THE PROCEEDINGS OF THE PHYSICAL SOCIETY

Section A

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Section A

VOL. 62, PART 12 1 December 1949 No. 360 A

Radiative Reaction and Damping in Scattering

BY J. HAMILTON
Manchester University

Communicated by L. Rosenfeld ; MS. received 14th June 1949

ABSTRACT. The higher order radiative corrections and the damping corrections to the cross section for the scattering of an electron by an electrostatic potential are investigated. It is shown that the perturbation treatment of the radiative corrections at high frequencies joins on to the Bloch-Nordsieck solution for low frequencies. The damping effect arising from the low frequency photons is shown to be negligible, and a modified damping equation is proposed which gives a smooth transition to the damping correction in passing from high to low frequency photons.

§ 1. INTRODUCTION AND SUMMARY

THE emission of low frequency photons during the scattering of an electron by an electrostatic potential has been described by Bloch and Nordsieck (1938), Pauli and Fierz (1938) and others. The method used by these authors suffers from the disadvantage that the upper limit, K, to the low frequency range considered enters into the final result. The simultaneous emission of high frequency photons can be treated by the ordinary perturbation methods, but the defect remains, for the frequency K dividing the 'low' from the 'high' frequency photons appears in the cross sections.

It might seem that radiation damping theory could resolve this difficulty. The damping due to the high frequency photons has been discussed by Bethe and Oppenheimer (1946), and it is shown below that the damping due to the low frequency photons is negligible. The resultant total damping effect, though of the right sign, has the wrong dependence on the scattering potential V to cancel the undesirable effect.

The arbitrary dependence of the cross sections on K is connected with the fact that, as a result of the Bloch–Nordsieck transformation, the 'free' photon states are modified due to the presence of the electron. At the same time the total Hamiltonian has been separated into an electron and a field part, and the electron part contains a portion of the electron's self energy. Dancoff (1939) studied the corresponding modified 'free' electron and photon states for the high frequency region, but he obtained expressions containing high frequency divergencies. Lewis (1948), Epstein (1948), and Koba and Tomonaga (1948) have shown how Dancoff's results can be made to give finite expressions by subtracting the electron's self energy. The resultant correction to the cross section for scattering without the emission of radiation, which we will call the

radiative reaction effect, is difficult to calculate. It will, however, be shown in the first part of this paper that this reactive effect for the high frequency photons has a complementary dependence on the dividing frequency K to that of the low frequency photon component, so that K itself drops out of the cross sections. Further, it is possible to make a rough estimate of the size of the correction to the cross section, and it is seen that the correction is small for a wide range of problems.

The question now arises as to whether there is also a damping correction to the cross section. The mass and charge renormalizing methods have actually enabled the finite, and significant, parts of the matrix elements arising from higher order virtual transitions to be calculated. At the same time they have renormalized the initial and final wave functions. The resultant total matrix elements could be used in a perturbation calculation of the cross sections. However, damping theory goes further in ensuring that the transition probabilities are normalized, so that the rate of decrease of the initial state equals the rate of increase of all the final states. Alternately, in the steady state solution, that particular solution representing the initial state plus outgoing final states is chosen. It thus seems that, even with the new matrix elements resulting from the renormalizing method, damping may give a further correction in calculating the cross sections.

In § 4 the damping effects are investigated using the Bloch–Nordsieck method for low frequency photons, and the ordinary perturbation method (without Dancoff's modified states) for high frequency photons. It appears that the low frequency photons contribute practically no damping, a fact which might have been guessed from the small energy and momentum emitted in the low frequency range. The separation of the Hamiltonian into an electron and a photon component is, in fact, the real cause.

The damping due to the high frequency photons, however, is not satisfactory because the dividing frequency K again appears in the result. In § 5 a possible method of avoiding this difficulty is discussed. It is, after all, inconsistent to treat the damping due to the high frequency photons without using Dancoff's modified states. The consequent treatment of the damping for transitions between these new 'free' electron and photon states has not been completely investigated. It is probable that the infinities which are neglected in obtaining the damping solution will be replaced by small finite corrections. However, some considerations are made on the effect of the lower frequency photons, and a modified form of damping equation is given in which the dependence of the damping correction on the lower limit of the high frequency photon range is eliminated.

In the Appendix the subtraction method due to Heitler and Peng (1942) is applied to the low frequency photon problem. It is shown that, unless the Bloch–Nordsieck transformation is used, the method is unsatisfactory, as it implies that the low frequency photons produce very strong damping of the electron's motion.

Finally it is worth emphasizing that the satisfactory solutions which have been obtained in the new representations seem to imply some special property of this representation. It may of course be possible, by calculating all the higher order effects, to show that these solutions are, in some way, equivalent to those calculated in any other representation, as would be the case with a system of a finite number of degrees of freedom.

§ 2. THE RADIATIVE REACTION

The Bloch–Nordsieck transformation can be expressed conveniently by Jost's (1947) formalism. The Hamiltonian of the system of electron, scatterer and radiation can be written

$$H=(\boldsymbol{\alpha}\,.\,\mathbf{p})+\beta m+\Sigma_\nu(\boldsymbol{\alpha}\,.\,\mathbf{a}_\nu)[q_\nu\exp\{i(\boldsymbol{\varkappa}_\nu\,.\,\mathbf{x})\}+\text{complex conjugate}]+V(\mathbf{x})+\Sigma_\nu n_\nu\kappa_\nu, \qquad \ldots\ldots (1)$$

where $\mathbf{p}$ is the electron's momentum, m its mass and $\boldsymbol{\alpha}, \beta$ the Dirac matrices. $V(\mathbf{x})$ is the electrostatic scattering potential, and n_ν, q_ν are the photon occupation numbers and amplitudes. $\mathbf{a}_\nu=\mathbf{e}_\nu\,.\,e\sqrt{(2\pi/L^3\kappa_\nu)}$, where $\mathbf{e}_\nu$ is a polarization vector and L the linear dimension of the enclosing periodic box. Also, $\hbar=c=1$.

The canonical transformations $F'=S_1^{-1}\,.\,F\,.\,S_1$, $F''=S_2^{-1}\,.\,F'\,.\,S_2$ are applied to the low frequency photons, defined by $\bar{\kappa}\leqslant\kappa_s\leqslant K$, where $\bar{\kappa}=2\pi/L$ is the least frequency allowed by the box. Using $S_1=\exp\{i\Sigma_s n_s(\boldsymbol{\varkappa}_s\,.\,\mathbf{x})\}$,

$$S_2=\exp\{\Sigma_s\eta_s\,.\,(\bar{q}'_s-q'_s)\},$$

where $\bar{q}$ is the Hermetian conjugate of q, and

$$\eta_s(\mathbf{p})=\frac{(\mathbf{p}'\,.\,\mathbf{e}_s)}{E'-(\mathbf{p}'\,.\,\boldsymbol{\varkappa}_s)/\kappa_s}(2\pi e^2)^{\frac{1}{2}}(\bar{\kappa}/\kappa_s)^{\frac{3}{2}} \qquad \ldots\ldots (2)$$

gives

$$\begin{aligned}H''(\mathbf{p}'',\mathbf{x}'',q_t,n''_s,\boldsymbol{\alpha})&\equiv H(\mathbf{p},\mathbf{x},q_t,q_s,\boldsymbol{\alpha})\\&=(\boldsymbol{\alpha}\,.\,\mathbf{p}''\left(+\beta m+V(\mathbf{x}''+\sqrt{2}\Sigma_s\frac{\partial\eta_s}{\partial\mathbf{p}}P''_s\right)+\Sigma_t n_t\kappa_t\\&\quad+\Sigma_t\{(\boldsymbol{\alpha}\,.\,\mathbf{a}_t)q_t\exp\{i(\boldsymbol{\varkappa}_t\,.\,\mathbf{x}')\}+\text{complex conjugate}\}\\&\quad+\Sigma_s\{\mathbf{p}''\,.\,\boldsymbol{\varkappa}_s\}[n''_s+\eta_s^2]/E''+\text{small terms}\ldots. \qquad \ldots\ldots (3)\end{aligned}$$

where $P''_s=(i/\sqrt{2})(q''_s-\bar{q}''_s)$, $\{\mathbf{p}\,.\,\boldsymbol{\varkappa}\}=E\kappa-(\mathbf{p}\,.\,\bar{\boldsymbol{\varkappa}})$, and $E=(m^2+p^2)^{\frac{1}{2}}$. t refers to high and s to low frequency photons. The omitted terms are $-\Sigma_s(\boldsymbol{\alpha}_1\,.\,\mathbf{a}_s)(q''_s+\bar{q}''_s)$, where $\boldsymbol{\alpha}_1=\boldsymbol{\alpha}-\mathbf{p}''/E''$, plus terms arising from the recoil of the electron during the emission of the low frequency photons. Solving the wave functions for H'' omitting V, and transforming back to the H' system, gives, for example, $\phi_{(in)}=\phi_i\,.\,\Pi_s h_{n_s}(Q'_s-\sqrt{2}\,.\,\eta_s(\mathbf{p}_i))$, where ϕ_i is the wave function of an electron of momentum $\mathbf{p}_i$, while $h_n(Q_s)$ is the free photon wave function, and $Q'_s=(q'_s+\bar{q}'_s)/\sqrt{2}$. Then, using the Born approximation, the matrix element for scattering and low frequency emission is $V_{ki}\Pi_s(m_s\mathbf{p}_k|n_s\mathbf{p}_i)$, where

$$(m_s\mathbf{p}_k|n_s\mathbf{p}_i)=\exp\{-w^2(k,i)/2\}\sqrt{(m_s!\;n_s!)}\sum_{\mu=0}^{\infty}\frac{[w(k,i)]^{m_s-n_s+2\mu}\,.\,(-)^\mu}{(n_s-\mu)!\;\mu!\;(m_s-n_s-\mu)!}, \qquad \ldots\ldots (4)$$

with $w(k,i)=\eta_s(\mathbf{p}_k)-\eta_s(\mathbf{p}_i)$ and $1/\sigma!=0$, if $\sigma<0$ (cf. Pauli and Fierz 1938). V_{ki} is the potential matrix element.

The term $-(\boldsymbol{\alpha}_1\,.\,\mathbf{a}_s)(q''_s+\bar{q}''_s)$ gives further low frequency photon transitions, such as two photon emission, scattering etc. The corresponding transition probabilities are of the order $\rho_k|V_{ki}|^2(e^2K/m)^N$, where N is the number of photons emitted or absorbed and ρ_k the state density. This effect is negligible if $K\ll m$, while the omitted recoil terms are small provided $K\ll|\mathbf{p}|$, the electron's momentum.

For a finite periodic box the probability of scattering from $\mathbf{p}$ to $\mathbf{q}$ without the emission of any photons is, approximately,

$$\rho_q |V_{qp}|^2 \exp\left\{-\frac{2}{3\pi}\frac{(\mathbf{p}-\mathbf{q})^2}{m^2 \log(K/\kappa)}\right\}.$$

Other transition probabilities, such as that for the emission of any number of low frequency photons κ_s, such that $\kappa_s \leqslant K' < K$, will depend on K in a similar fashion. Increasing K to $K+dk$ gives (for relativistic or slow electrons)

$$d\sigma/\sigma = -(e^2/4\pi^2) K\, dk \int d\Omega_\kappa \{(\mathbf{P}-\mathbf{Q})^2 - (\mathbf{P}-\mathbf{Q}\,.\,\hat{\boldsymbol{\varkappa}})^2\}, \qquad \ldots\ldots (5)$$

where $\mathbf{P} = \mathbf{p}/\{\mathbf{p}\,.\,\mathbf{K}\}$, and $\hat{\boldsymbol{\varkappa}}$ is a unit vector which is to be integrated over $d\Omega_\kappa$. $d\sigma/\sigma$ is the relative change in the cross section concerned. Treating high frequency photons by the conventional perturbation methods also leads to (5) for the relative change in cross section on varying K.

The arbitrary factor K enters the cross sections because the initial and final wave functions for the low frequency photons are, in the H' system, $h_n(Q'_s - \sqrt{2}\,.\,\eta_s)$, while those for the high frequency photons are $h_n(Q'_t)$. Dancoff (1939) has tried to rectify this asymmetry by using perturbation theory to find the field of high frequency photons which accompanies a freely moving electron, due to the effect of the electron's charge on the vacuum. In fact he tries to find that eigenfunction of

$$H = (\boldsymbol{\alpha}\,.\,\mathbf{p}) + \beta m + \Sigma_t \{(\boldsymbol{\alpha}\,.\,\mathbf{a}_t) q_t \exp\{i(\boldsymbol{\varkappa}_t\,.\,\mathbf{x})\} + \text{complex conjugate}\} + \Sigma_t n_t \kappa_t,$$

which reduces to a free electron wave function as $e^2 \to 0$. Using such wave functions he calculates the change in the cross section for radiationless scattering. His calculations have been elaborated and corrected by Ito, Koba and Tomonaga (1948), who get

$$\left(\frac{\delta\sigma}{\sigma_0}\right)_{e^2} = -\frac{3e^2}{4\pi}\int^\infty \frac{dl}{l} + \frac{3e^2}{2\pi}\frac{m^2(\mathbf{p}-\mathbf{q})^2}{E^2\{E^2 + (\mathbf{p}\,.\,\mathbf{q}) + m^2\}} \cdot \int^\infty \frac{dk}{k} + \text{finite terms.} \qquad \ldots\ldots (6)$$

The integration over l refers to electron momenta, and is a vacuum polarization effect, while the k integration arises from virtual photon processes. It is assumed that the scattering potential is electrostatic. Lewis (1948), Epstein (1948) and Koba and Tomonaga (1948) have shown how the infinite terms can be subtracted. The latter authors write the total Hamiltonian in the form

$$H = \int \bar{\psi}\{(\boldsymbol{\alpha}\,.\,\mathbf{p}) + \beta m\}\psi\, d\mathbf{x} + H_{\text{radiation}} + H_{\text{interaction}}$$

$$= \int \bar{\psi}\{(\boldsymbol{\alpha}\,.\,\mathbf{p}) + \beta \overset{0}{m}\}\psi\, d\mathbf{x} + H_{\text{rad.}} + \overset{0}{H}_{\text{int.}},$$

where $\overset{0}{m} = m + \delta m$ and $\overset{0}{H}_{\text{int.}} = H_{\text{int.}} - \delta m \int \bar{\psi}\beta\psi\, d\mathbf{x}$. δm is the electromagnetic self energy of the electron (to order e^2) as calculated by Weisskopf (1939) viz.,

$$\delta m = \frac{3}{2\pi} e^2 m \{\log([k_\infty + (k_\infty^2 + m^2)^{\frac{1}{2}}]/m) - \tfrac{1}{6}\},$$

where $k_\infty \to \infty$. $\overset{0}{m}$ is thus the observed electron mass, to order e^2. The $\overline{\psi}$, ψ are quantized electron waves, $H_{\text{rad.}} = \Sigma_\nu n_\nu \kappa_\nu$, while

$$H_{\text{int.}} = e \int \overline{\psi} \boldsymbol{\alpha} \psi \,.\, \Sigma_\nu \{\mathbf{a}_\nu q_\nu \exp\{i(\boldsymbol{\kappa}_\nu \,.\, \mathbf{x})\} + \text{complex conjugate}\} \, d\mathbf{x}$$

$$+ \frac{e^2}{2} \int\int \overline{\psi}(\mathbf{x}) \psi(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{x}'|} \overline{\psi}(\mathbf{x}') \psi(\mathbf{x}') \, d\mathbf{x} \, d\mathbf{x}'.$$

The mass renormalizing term $-\delta m \,.\, \int \overline{\psi} \beta \psi \, d\mathbf{x}$ contributes a correction to the cross section for the scattering from $\mathbf{p}$ to $\mathbf{q}$. For example, this δm term creates an electron pair, denoted by $\mathbf{q}$, $(-\mathbf{q})^+$, and subsequently the potential V annihilates $\mathbf{p}$ and $(-\mathbf{q})^+$. The resultant correction is

$$\left(\frac{\delta\sigma}{\sigma_0}\right)_m = -\delta m \frac{\overset{0}{m}(\mathbf{p} - \mathbf{q})^2}{\overset{0}{E}^2 \{\overset{0}{E}^2 + (\mathbf{p} \,.\, \mathbf{q}) + m^2\}},$$

where $\overset{0}{E} = (\overset{0}{m}{}^2 + \mathbf{p}^2)^{\frac{1}{2}}$. Using (6) and the value of δm above, it is clear that the infinite term in k no longer appears in $(\delta\sigma/\sigma_0)_{e^2} + (\delta\sigma/\sigma_0)_m$. Koba and Tomonaga also discuss how the vacuum polarization infinity can be counteracted by redefining an observed potential $\overset{0}{V}(\mathbf{x})$.

It is now of interest to see whether the remaining finite radiative reaction will eliminate the difficulties arising from the arbitrariness of K in the Bloch–Nordsieck transformation. When the Bloch–Nordsieck transformation is applied to photons such that $\kappa_s < K$, Dancoff's procedure will have to be modified.

Following Ito, Koba and Tomonaga (1948) we see that the free electron state Ψ'_p interacts to order e with the states $\Psi'^{(\mathrm{i})}_p(\mathbf{p} - \mathbf{k}, \tilde{\mathbf{k}})$, $\Phi^{(\mathrm{i})}_p(\mathbf{p}, \mathbf{l}, (-\mathbf{l} + \mathbf{k})^+, -\tilde{\mathbf{k}})$, where $\mathbf{l}$ indicates an electron, $(-\mathbf{l})^+$ a positron and $\tilde{\mathbf{k}}$ a photon. The wave function $\Phi^{(\mathrm{i})}_p$ is given by an operator of the type $\lambda^-_{\mathbf{l}-\mathbf{k}}(\boldsymbol{\alpha} \,.\, \mathbf{e})\lambda^+_{\mathbf{l}} / \{E(\mathbf{l}) + E(\mathbf{l} - \mathbf{k}) + \mathbf{k}\}$, where $\lambda^{\pm}_{\mathbf{p}} = \{E(\mathbf{p}) \pm (\boldsymbol{\alpha} \,.\, \mathbf{p}) \pm \beta m\}/2E(\mathbf{p})$, and $E(\mathbf{p}) = (m^2 + \mathbf{p}^2)^{\frac{1}{2}}$. This will be practically unaltered in the form $\lambda^-_{\mathbf{l}-\boldsymbol{\kappa}_s}(\boldsymbol{\alpha}_1 \,.\, \mathbf{e}_s)\lambda^+_{\mathbf{l}} / \{E(\mathbf{l}) + E(\mathbf{l} - \boldsymbol{\kappa}_s) + \kappa_s\}$, where $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha} - \mathbf{p}/E$ replaces $\boldsymbol{\alpha}$ for photons κ_s such that $\bar{\kappa} \leqslant \kappa_s < K$. In fact, $\lambda^-_{\mathbf{l}-\mathbf{k}}(\mathbf{l}/E(\mathbf{l}))\lambda^+_{\mathbf{l}}$ gives a contribution $O(K/E(\mathbf{l}))$, while $\lambda^-_{\mathbf{l}-\mathbf{k}}(\boldsymbol{\alpha} \,.\, \mathbf{e})\lambda^+_{\mathbf{l}}$ gives $O(1)$. On the other hand, the state $\Psi'^{(\mathrm{i})}_p(\mathbf{p} - \mathbf{k}, \tilde{\mathbf{k}})$ does not arise for $\kappa_s < K$, as there are no transitions of order e between positive energy states when the Bloch–Nordsieck wave functions are used. The states of order e^2 are reached either through $\Psi'^{(\mathrm{i})}_p$ or $\Phi^{(\mathrm{i})}_p$, or directly from Ψ'_p through the Coulomb interaction between electrons. This interaction is unaltered by the Bloch–Nordsieck transformation. Finally, there is the renormalization connection for the wave function Ψ'_p. To order e^2 this arises from $|\Psi'^{(\mathrm{i})}_p|^2$ and $|\Phi^{(\mathrm{i})}_p|^2$ only.

The amount by which the finite reactive terms will be modified on altering the dividing frequency from K to $K + dk$ can now easily be estimated. The important effects arise from $\int d\mathbf{k} (\overline{\Psi}'^{(\mathrm{i})}_q(\mathbf{q} - \mathbf{k}, \tilde{\mathbf{k}}) V \Psi'^{(\mathrm{i})}_p(\mathbf{p} - \mathbf{k}, \tilde{\mathbf{k}}))$, and the renormalization correction $\Psi'^{(\mathrm{ii})}_{p(n)} = -\frac{1}{2} \int d\mathbf{k} |\Psi'^{(\mathrm{i})}_p(\mathbf{p} - \mathbf{k}, \tilde{\mathbf{k}})|^2$, giving $(\overline{\Psi}'^{(\mathrm{ii})}_{q(n)} V \Psi'_p)$ and $(\overline{\Psi}'_q V \Psi'^{(\mathrm{ii})}_{p(n)})$. The first gives, in the range K, $K + dk$,

$$d\left(\frac{\delta\sigma}{\sigma_0}\right)_A =$$

$$\frac{e^2}{2\pi^2} \cdot \int_K^{K+dk} d\mathbf{k} \,.\, \sum_{\text{pol.}} \frac{\text{Spur}\{\lambda_{\mathbf{p}} V \lambda^+_{\mathbf{q}}(\boldsymbol{\alpha} \,.\, \mathbf{e}) \lambda^+_{\mathbf{q}-\mathbf{k}} V \lambda^{\pm}_{\mathbf{p}-\mathbf{k}}(\boldsymbol{\alpha} \,.\, \mathbf{e})\}}{\{E(\mathbf{p}) - E(\mathbf{p} - \mathbf{k}) - k\}\{E(\mathbf{q}) - E(\mathbf{q} - \mathbf{k}) - k\} k \,.\, \text{Spur}\{\lambda^+_{\mathbf{p}} V \lambda^+_{\mathbf{q}} V\}}.$$

$\Sigma_{\text{pol.}}$ indicates the sum over photon polarizations. Expanding in powers of K/m or K/E, both of which are very small, gives

$$d(\delta\sigma/\sigma_0)_A \simeq (e^2/2\pi^2)K dk \int d\Omega_\kappa \{(\mathbf{P}\,.\,\mathbf{Q}) - (\mathbf{P}\,.\,\hat{\mathbf{x}})(\mathbf{Q}\,.\,\hat{\mathbf{x}})\}$$

where $\mathbf{P}$, $\mathbf{Q}$ are defined above. Similarly the renormalization terms give

$$d\left(\frac{\delta\sigma}{\sigma_0}\right)_n = -\frac{1}{2}\left(\frac{e^2}{4\pi^2}\right)\int_K^{K+dk} d\mathbf{k}\,.\sum_{\text{pol.}} \frac{\text{Spur}\,\{\lambda_{\mathbf{p}}^+(\boldsymbol{\alpha}\,.\,\mathbf{e})\lambda_{\mathbf{p}-\mathbf{k}}^+(\boldsymbol{\alpha}\,.\,\mathbf{e})\}}{\{E(\mathbf{p}) - E(\mathbf{p}-\mathbf{k}) - k\}^2\,.\,k} + \langle \mathbf{p}, \mathbf{q}\rangle.$$

where $\langle \mathbf{p}, \mathbf{q}\rangle$ is the same expression with $\mathbf{p}$ exchanged for $\mathbf{q}$. Thus

$$d(\delta\sigma/\sigma_0)_n \simeq -(e^2/4\pi^2)K dk \int d\Omega_\kappa \{\mathbf{P}^2 + \mathbf{Q}^2 - (\mathbf{P}\,.\,\hat{\mathbf{x}})^2 - (\mathbf{Q}\,.\,\hat{\mathbf{x}})^2\}.$$

Hence

$$d(\delta\sigma/\sigma_0)_A + d(\delta\sigma/\sigma_0)_n = -(e^2/4\pi^2)K dk \int d\Omega_\kappa \{(\mathbf{P}-\mathbf{Q})^2 - (\mathbf{P}-\mathbf{Q}\,.\,\hat{\mathbf{x}})^2\} + \text{small terms} \quad \ldots\ldots (7)$$

The small terms in (7) are of the order of K/m times the main contribution. For non-relativistic electrons (7) reduces to $-(2e^2/3\pi)\{(\mathbf{p}-\mathbf{q})^2/m^2\}dk/K$.

The other terms $(\delta\sigma/\sigma_0)$ of order e^2 which arise from virtual photon processes are of the same general form as $(\delta\sigma/\sigma_0)_A$ (cf. Ito, Koba and Tomonaga 1948). The spur in the numerator is of the same order as the spur in the denominator, while the denominator also contains the factor k and two energy differences. In order to get a result of order dk/K it is necessary for both energy differences to be of order K, and this can only arise from transitions between positive energy states, and hence only from $\Psi_p^{(i)}(\mathbf{p}-\mathbf{k}, \tilde{\mathbf{k}})$, or $\Psi_q^{(i)}(\mathbf{q}-\mathbf{k}, \tilde{\mathbf{k}})$.

Finally, there are the vacuum polarization terms and the self energy to be considered. The vacuum polarization terms are only influenced by the Bloch–Nordsieck transformation when $|\mathbf{p}-\mathbf{q}| < K$, and then the effect is negligible for the general reasons stated above. The effect on the self energy can be estimated from the self energy separated off by the Bloch–Nordsieck method. For a non-relativistic electron this is of the order of $e^2(p/m)^2K$. The resultant change in the infinite balancing contribution $(\delta\sigma/\sigma_0)_m$ is of the order $\{(\mathbf{p}-\mathbf{q}^2, m^2\}(p, m)^2K, m$, which is thus K/m times the main terms considered. The situation is similar for a relativistic electron.

Comparing (7) and (5) it follows that an increase in K decreases the cross section as calculated by Bloch and Nordsieck, and increases, by the same amount, the high frequency radiative correction so that $\partial\sigma/\partial K = 0$. Thus, to the order e^2 considered, K no longer enters the final results. The only limitation is that $K \ll m$. The small corrections in K/m can doubtless be eliminated by a more exact treatment. It is also interesting to note that the radiative reaction would give an infra-red divergence in $(\delta\sigma/\sigma_0)_{e^2}$, if the Bloch–Nordsieck transformation were not applied to the low frequency photons.

It seems to be almost certain that the same treatment will avoid the arbitrariness due to K in other problems, such as Jost's (1947) treatment of Compton scattering. Finally it should be mentioned that Tati and Tomonaga (1948) have proposed a relativistic generalization of the Bloch–Nordsieck transformation, using the subtraction methods. However, they have not published any precise details of the method to date.

§ 3. THE MAGNITUDE OF THE RADIATIVE CORRECTION

In this section only that part of the reaction which is due to virtual photon processes will be considered. The remainder is the vacuum polarization effect, and will not be discussed here. The exact evaluation of the various terms in $(\delta\sigma/\sigma_0)$ would be tedious, but it is readily possible to make a rough estimate of the size of the effect, following the formulae of Ito, Koba and Tomonaga. For lower frequencies k such that $K<k\ll m$, the prominent terms are given in the preceding section. For $k\gg E$, the electron's energy, those terms which do not converge are of the order of $e^2\{\log(k_\infty/E)+O(1)\}$ where k_∞ is the upper limit of the virtual photon integration, which tends to infinity. Subtracting a similar term, giving the effect of the electron self energy, leaves the finite term $e^2\theta(1)$. Now it is only necessary to note that the various integrands occurring in $(\delta\sigma/\sigma_0)$ have the form $O(e^2/E)$ in the region $k\sim E$. It follows that a rough approximation to the finite radiative reaction is $(\delta\sigma/\sigma_0)_{e^2}\sim O(e^2\log(E/K))$, where E is the relativistic energy of the electron, while K is the frequency dividing the low from the high frequency photons (this formula overestimates the effect for the lower frequencies when the electron is non-relativistic).

If we can choose K such that both $K\ll m$ and $e^2\log(E/K)\ll 1$ are satisfied, then the radiative reaction is a small effect (to order e^2), and it is sufficient to use perturbation theory for its computation. The limitation on E is not of practical importance.

It is of interest to see if anything can be deduced about the physical interpretation of the modified cross section analogous to the well-known interpretation of the Bloch–Nordsieck results. For a non-relativistic electron it is clear that if photons of frequency less than K are neglected, the reactive term for an appreciable energy range behaves like

$$-(2/3\pi)\sigma_0 e^2\{(\mathbf{p}-\mathbf{q})^2/m^2\}\int_K dk/k.$$

Thus, in this range, the cross section for radiation free scattering is decreased by an amount which is just the total cross section for the emission of one photon. (The emission of more than one photon would have to be compared with terms in e^4 in $\delta\sigma$.)

About the higher frequency part ($k\sim m$) of the reaction little can be said without calculating the exact form of $\delta\sigma$. However, the ratio of the total probability of the emission of one photon of frequency greater than K to the scattering probability is, for a relavivistic electron, of the order of $e^2\log(E/K)$. This is, according to the rough estimate made above, just the order of $(\delta\sigma/\sigma_0)_{e^2}$. Nevertheless the simple interpretation of the correction to the scattering cross section as being the total probability of emission of a photon must break down for two reasons: (i) there are high frequency terms in the radiative reaction which cannot be compared with any real emission; (ii) the emission of a high frequency photon $k\sim|\mathbf{p}|$ no longer leaves the electron's motion practically unaltered.

§ 4. DAMPING EFFECTS

The damping effects can be readily found on introducing the Bloch–Nordsieck transformation for $\bar{\kappa}<\kappa_s<K$, where $K\ll m$, and $\bar{\kappa}=2\pi/L$ is the least frequency allowed by the finite size of the periodic box. First we will consider the case of a slowly moving electron, with momentum $\mathbf{p}$ satisfying $|\mathbf{p}|\ll m$, so that all the

emitted photons can be treated by the Bloch–Nordsieck method. The simplest form of the damping equation in linear momentum representation is thus

$$U_{mq,np} = H_{mq,np} - i\pi \Pi_s \Sigma_{n'_s} \int H_{m_s q, n'_s p'} \rho_{p'} U_{n'_s p', np}, \qquad \ldots\ldots (8)$$

where p, q refer to the initial and final electron states, while n_s, m_s are the numbers of photons of type κ_s in the initial and final states respectively. Also $H_{mq,np} = \Pi_s (m_s \mathbf{q} | n_s \mathbf{p}) \cdot V_{qp}$, where V_{qp} is the matrix element of the scattering potential and $(m_s \mathbf{q} | n_s \mathbf{p})$ is given by (4). $\rho_{p'}$ is the state density function. Letting $e^2 \to 0$, equation (8) gives

$$U_{q,p} = V_{qp} - i\pi \int V_{qp'} \rho_{p'} U_{p'p}. \qquad \ldots\ldots (9)$$

It has been shown in another paper (Hamilton 1949 a) that (9) gives an accurate description of the collision (when $e \to 0$) only when V is small. Precisely, if V_{qp} is not strongly dependent on the direction of $\mathbf{p}$ or $\mathbf{q}$, the condition is

$$\rho_p |V_{pq}| \ll 1. \qquad \ldots\ldots (10)$$

This condition will be assumed. (10) is also a necessary condition for the validity of the Born approximation; and it must be expected that the equations (8) in linear momentum representation, which essentially implies the Born approximation, will only be valid under this condition. Equation (8) can be solved by making the substitution $U_{mq,np} = \Pi_s (m_s \mathbf{q} | n_s \mathbf{p}) \cdot \mathbf{U}_{qp}$, where the low frequency photons no longer appear in $\mathbf{U}_{qp}$. Using the relation (cf. Pauli and Fierz 1938)

$$\Sigma_{n'_s} (m_s \mathbf{q} | n'_s \mathbf{p}')(n'_s \mathbf{p}' | n_s \mathbf{p}) = (m_s \mathbf{q} | n_s \mathbf{p}) \qquad \ldots\ldots (11)$$

the solution is

$$\mathbf{U}_{q,p} = V_{qp} - i\pi \int V_{qp'} \rho_{p'} \mathbf{U}_{p'p}. \qquad \ldots\ldots (12)$$

This solution is obtained by neglecting the recoil and the energy of the low frequency photons. The difficulties arising out of neglecting the energy of the low frequency photons, which were pointed out by Pauli and Fierz (1938), will not be discussed. That part of the interaction between the electron and the radiation field which is neglected in making the Bloch–Nordsieck transformation, and which was discussed in §2, will give a small correction to (12) of the relative order of $(e^2 K/m)^2$.

Equation (12) is equivalent to equation (9), so that the low frequency photons give rise to a negligible damping effect. Further, what small effect there is arises from the photons at the higher end of the frequency range K. From a correspondence principle argument it is not surprising that the large or infinite (case $\bar{\kappa} \to 0$) number of low frequency photons produces no appreciable damping effect. The emission of photons of very low frequency is almost a continuous process, so that classical theory can be used with little error. This gives $E(\kappa)\, d\kappa = (2e^2/3\pi) \cdot (\mathbf{v} - \mathbf{w})^2 \cdot d\kappa$ for the emitted energy distribution $E(\kappa)\, d\kappa$, where $\mathbf{v}$, $\mathbf{w}$ are the initial and final velocities and κ is the frequency of the emitted radiation. This relation holds so long as the frequency κ is less than the reciprocal of the 'collision time'. The reaction due to the emitted radiation must depend on the energy taken from the electron (and scatterer) by the radiation. However, as the integrated energy depends linearly on the range of integration, this reaction is negligible in some small frequency range containing the infinity of photons. As the frequency range increases, the reaction is no longer vanishingly small,

and some damping effect will arise. From the quantum mechanical viewpoint the absence of damping is due to the separation of the electron and low frequency photon variables in the Hamiltonian. This separation is, of course, complete only in the case of infinitesimally small quanta.

The damping of collisions in which high frequency photons may be emitted has been treated by Bethe and Oppenheimer (1946). For convenience they assume that only one photon is emitted, and consequently their results are only valid under the condition $e^2\log(T/K)\ll 1$, where T is the initial kinetic energy of the electron and K is the lower limit of frequency of the photons considered. If at the same time $K\ll m$, photons of frequency less than K can be treated by the Bloch–Nordsieck method. The generalization of equation (8) (omitting an unimportant term) is

$$\left.\begin{aligned} U_{mq,np} &= H_{mq,np} - i\pi\Pi_s\Sigma_{n_s'}\int H_{m_sq,n_s'p'}\rho_{p'}U_{n_s'p',n_sp} \\ &\quad - i\pi\Pi_s\Sigma_{n_s'}\int H_{m_sq,n_s'p'j'}\rho_{p'}\rho_{j'}U_{n_s'p'j',n_sp} \\ U_{mqj,np} &= H_{mqj,np} - i\pi\Pi_s\Sigma_{n_s'}\int H_{m_sqj,n_s'p'}\rho_{p'}U_{n_s'p',n_sp} \\ &\quad - i\pi\Pi_s\Sigma_{n_s'}\int H_{m_sqj,n_s'p'j'}\rho_{p'}\rho_{j'}U_{n_s'p'j',n_sp} \end{aligned}\right\} \quad \ldots\ldots (13)$$

n, m refer to low frequency quanta and j, j' to high frequency quanta. The integrations are to be taken over states whose energy equals the initial energy, and $\rho_{p'}$, $\rho_{j'}$ are the density functions. The integrations over the high frequency photons go from K to T.

Substituting $H_{mq,np}=\Pi_s(m_s\mathbf{q}|n_s\mathbf{p})\,.\,V_{qp}$, $H_{mqj,np}=\Pi_s(m_s\mathbf{q}|n_s\mathbf{p})\,.\,H_{qj,p}$, where $H_{qj,p}$ is the matrix element for the emission of the high frequency photon j, and, further, substituting $U_{mq,np}=\Pi_s(m_s\mathbf{q}|n_s\mathbf{p})\,.\,\mathbf{U}_{qp}$, $U_{mqj,np}=\Pi_s(m_s\mathbf{q}|n_s\mathbf{p})\,.\,\mathbf{U}_{qj,p}$, and using (11), separates off the low frequency photon part and gives the equations

$$\left.\begin{aligned} \mathbf{U}_{q,p} &= V_{qp} - i\pi\int H_{q,p'j'}\rho_{p'}\rho_{j'}\mathbf{U}_{p'j',p} - i\pi\int V_{qp'}\rho_{p'}\mathbf{U}_{p'p} \\ \mathbf{U}_{qj,p} &= H_{qj,p} - i\pi\int V_{qp'}\rho_{p'}\mathbf{U}_{p'j,p} - i\pi\int H_{qj,p'}\rho_{p'}\mathbf{U}_{p'p} \end{aligned}\right\} \quad \ldots\ldots (14)$$

Solving (14) by the method of reciprocal kernels, and using the relations $\rho_p|V_{pq}|\ll 1$, $e^2\log(T/K)\ll 1$, gives

$$\begin{aligned} \mathbf{U}_{q,p} \simeq \{V_{qp} - i\pi\int V_{qp'}\rho_{p'}V_{p'p}\} - i\pi\int H_{q,p'j'}\rho_{p'}\rho_{j'}H_{p'j',p} \\ - \pi^2\int H_{q,p'j'}\rho_{p'}\rho_{j'}V_{p'p''}\rho_{p''}H_{p''j',p} - \pi^2\int V_{qp''}\rho_{p''}H_{p'',p'j'}\rho_{p'}\rho_{j'}H_{p'j',p} \\ - \pi^2\int H_{q,p'j'}\rho_{p'}\rho_{j'}H_{p'j',p''}\rho_{p''}V_{p''p} + \text{smaller terms.} \qquad \ldots\ldots (15) \end{aligned}$$

The first term in brackets on the right-hand side of (15) could have been obtained directly from (9), so that the remaining three terms give the radiation damping effect.

The magnitude of the damping correction is obtained by evaluating $|\mathbf{U}_{q,p}|^2$. For simplicity it is assumed that V_{qp} and $H_{qj,p}$ are real, a statement which is certainly true for the large values of $H_{qj,p}$ arising from photons j, whose frequency is near the dividing frequency. It is further assumed that V_{qp} is almost independent of the directions of $\mathbf{p}$ and $\mathbf{q}$. Then

$$|\mathbf{U}_{q,p}|^2 \simeq |V_{qp}|^2 - O\{e^2\log(T/K)\,.\,\rho_p{}^2|V|^4\}. \qquad \ldots\ldots (16)$$

The damping given by (16) has the correct sign but has the wrong dependence on $\rho|V|$ to cancel the arbitrariness due to the presence of K in the transition probabilities for the low frequency component. In fact this damping term is smaller than the radiative reaction discussed in §§ 2 and 3 by a factor of order $\{\rho|V|\}^2$.

It is troublesome that the factor $e^2\log(T/K)$ enters (16). Clearly a similar method would introduce factors of the type $\{e^2\log(T\,K)\rho^2|V|^2\}^N$ when states containing N high frequency photons are considered. Again, it seems that there is a term in the cross section depending on an arbitrary frequency K. It is not possible, in principle, to omit the damping correction, for as was pointed out in the introduction, damping and radiative reaction are distinct. The latter results from an accurate calculation of the matrix elements, while the former comes from an accurate deduction of the transition probabilities for any given matrix elements.

§ 5. THE MODIFIED DAMPING EQUATIONS*

The difficulty arising out of the presence of the lower limit, K, of the 'high' frequency photon spectrum in the damping term cannot be avoided by any consideration of the damping arising from 'low' frequency photons ($\kappa_s<K$), as this effect is negligible. Indeed, the smallness of the damping for $\kappa_s<K$ suggests that those photons with frequency a little greater than K should also give rise to a small damping effect. It would also be reasonable to guess that on writing the damping equations in terms of the new wave functions which were introduced in § 2, the undesirable dependence of the results on K will disappear.

Let Ψ_p be the wave function of an electron in state $\mathbf{p}$ (in linear momentum representation) unaccompanied by any photon field, $\Psi_{p\tilde{k}}$ that for an electron $\mathbf{p}$ and a photon $\tilde{\mathbf{k}}$ with no accompanying photons, etc. Then the corresponding wave functions Ψ which allow for the influence of the electron on the transverse component of the electromagnetic field are (cf. § 2):

$$\Psi_p=\Psi_p.\{1-O(e^2)\}+\Sigma_k\Psi_p^{(\mathrm{i})}(\mathbf{p}-\mathbf{k},\tilde{\mathbf{k}})$$
$$+\Sigma_{k,l}\Phi_p^{(\mathrm{i})}(\mathbf{p},\mathbf{l},(-\mathbf{l}+\mathbf{k})^+,-\tilde{\mathbf{k}})+\text{terms in } e^2\ldots.; \qquad \ldots\ldots (17a)$$

$$\Psi_{p\tilde{k}}=\Psi_{p\tilde{k}}\{1-O(e^2)\}+\Sigma_{k'}\Psi_{p\tilde{k}}^{(\mathrm{i})}(\mathbf{p}-\mathbf{k}',\tilde{\mathbf{k}},\tilde{\mathbf{k}}')+\Psi_{p\tilde{k}}^{(\mathrm{i})}(\mathbf{p}+\mathbf{k})$$
$$+\Sigma_l\Phi_{p\tilde{k}}^{(\mathrm{i})}[\mathbf{p},\mathbf{l},(-\mathbf{l}+\mathbf{k})^+]+\Sigma_{k',l}\Phi_{p\tilde{k}}^{(\mathrm{i})}[\mathbf{p},\mathbf{l},(-\mathbf{l}+\mathbf{k}')^+,\tilde{\mathbf{k}},-\tilde{\mathbf{k}}']$$
$$+\text{terms in } e^2\ldots. \qquad \ldots\ldots (17b)$$

The ordinary matrix element $H_{q,p\tilde{j}}$ for the absorption of a photon agrees with the matrix element $(\overline{\Psi}_q V\Psi_{p\tilde{j}})$ in terms of order e. They differ in the e^2 terms which occur in the latter, partly through higher order virtual transitions, and partly through the renormalization of the initial and final states.

Any attempt to write the damping matrix $\mathbf{U}$ in terms of the new representation given by Ψ_p, $\Psi_{p\tilde{i}}$, ... meets with the difficulty that $\mathbf{U}_{p'p}$ is only defined for $\mathbf{p}'$, $\mathbf{p}$ which have equal energy. Further, the introduction of the new representation is only possible if the various matrix elements are convergent, or if they can be made to converge by using the mass renormalization method. The damping equations (13) are obtained by neglecting several infinite terms arising from high

* I am obliged to Professor W. Heitler and Dr. S. T. Ma for letting me see the manuscript of a paper (1949) in which the modified form of the damping equation is considered. However, they do not treat the low frequency problem in any detail.

frequencies, and it is to be hoped that the renormalization method will reduce these infinite terms to small finite corrections.

Using a model in which the frequencies κ_t are limited to the range $K<\kappa_t<K'$, where $K' \ll$ the least of $|\mathbf{p}|$ or m ($\mathbf{p}$ being the initial momentum of the electron), the energy and momentum of the emitted photons can be neglected. Then the damping equation could be assumed to take the form

$$(\overline{\Psi}_q \mathbf{U} \Psi_p) = (\overline{\Psi}_q V \Psi_p) - i\pi \int (\overline{\Psi}_q V \Psi_{p'}) \rho_{p'} (\overline{\Psi}_{p'} \mathbf{U} \Psi_p)$$
$$- i\pi \int_K^{K'} dE_{\tilde{j}'} \int (\overline{\Psi}_q V \Psi_{p'\tilde{j}'}) \rho_{p'} \rho_{\tilde{j}'} (\Psi_{p'\tilde{j}'} \mathbf{U} \Psi_p) - \ldots \quad \ldots\ldots (18)$$

Writing $$I_{p'} = \Psi_{p'} \overline{\Psi}_{p'} + \Sigma_{\tilde{j}'} \Psi_{p'\tilde{j}'} \overline{\Psi}_{p'\tilde{j}'} + \Sigma_{\tilde{j}'\tilde{j}''} \Psi_{p'\tilde{j}'\tilde{j}''} \overline{\Psi}_{p'\tilde{j}'\tilde{j}''} + \ldots .$$

it is clear that it is terms arising from the second, third, ... terms in $\partial_{p'}$ which lead to the strong dependence of the damping, as calculated in § 4, upon the lower limit K.

Owing, however, to the replacement of the wave functions Ψ_p, $\Psi_{p\tilde{i}}$ by the $\mathbf{\Psi}_p$, $\mathbf{\Psi}_{p\tilde{j}}$ etc., it can be seen, on using (17), that

$$I_{p'} = u_{p'}(\mathbf{x}) \,.\, \Pi_i \delta(Q_i - Q_i') \,.\, \bar{u}_{p'}(\mathbf{x}') + \text{terms of relative order } (K/m).$$

$U_{p'}(\mathbf{x})$ is the wave function for an electron in state $\mathbf{p}'$, while $\mathbf{x}$, Q_i; $\mathbf{x}'$, Q_i' are the variables of integration occurring in the pairs of matrix elements. Using (17), it also follows that the largest part of the terms in e and e^2 arising in $I_{p'}$ cancel, leaving only terms of relative order (K/m).

Substituting $I_{p'}$ in equation (18), it follows that the strong dependence of the damping terms on K (through $\tilde{i}'$ etc.) is no longer present. This method is clearly a perturbation theory analogue of the Bloch–Nordsieck transformation. However, the modified form of the damping equation requires further investigation.

Finally, it should be noticed that the equation

$$U_{qp} = V_{qp} - i\pi \int V_{qp'} \rho_{p'} U_{p'p}$$

does not give an accurate description of the collision cross section when $e=0$. It was pointed out in an earlier paper (Hamilton 1949 a) that some extra terms have to be added to this equation to obtain the exact solution. This correction also would have to be incorporated in an accurate damping equation for 'bremsstrahlung'.

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APPENDIX

SUBTRACTION PROCEDURES

It is possible to solve the damping equations for the emission of a large number of low frequency photons without having performed the Bloch–Nordsieck transformation. The method fails because it gives a strong damping due to the low frequency photons which greatly influences the electron's motion. Two alternative subtraction methods (of the old kind) can be employed. The details are given briefly.

It is fairly simple to calculate the compound matrix elements for the emission or absorption of any number of low frequency photons. It will be assumed that the incident electron has a non-relativistic velocity, and that the momentum recoil can be neglected. The matrix elements for emission, for example, can be evaluated as follows. Since the photon recoil is to be neglected, there will be a number of emissions during which the electron momentum is approximately $\mathbf{p}_0$, then a scattering from $\mathbf{p}_0$ to $\mathbf{p}_F$ by the potential V, and finally further emissions. Only terms linear in V will be considered.

Using the interaction $-\{(\mathbf{p}.\mathbf{e}_s)/mc\}(q_s+\bar{q}_s)$ for emission or absorption, the emissions occurring before the scattering give a numerator of the form

$$\{-(e/mc)\sqrt{(2\pi\hbar^2c^2/L^3)}\}^{n_\lambda+n_\mu+\cdots}(\epsilon_\lambda^{n_\lambda}\epsilon_\mu^{n_\mu}\ldots)^{-\frac{1}{2}}(\mathbf{p}_0.\mathbf{e}_\lambda)^{n_\lambda}(\mathbf{p}_0.\mathbf{e}_\mu)^{n_\mu}\ldots\sqrt{(\mathbf{n}_\lambda!\,\mathbf{n}_\mu!\ldots)}$$

where n_λ photons of energy ϵ_λ, etc., have been emitted. The factorial term arises from the matrix elements of q_s. The corresponding energy denominator is easily calculated, using the fact that the order of emission of the individual photons varies. A typical term is

$$(-)^{n_\lambda+n_\mu+\cdots}1/\{\epsilon_\lambda(\epsilon_\lambda+\epsilon_\mu)(\epsilon_\lambda+\epsilon_\mu+\epsilon_\nu)\ldots.(n_\lambda\epsilon_\lambda+n_\mu\epsilon_\mu+\ldots.)\}.$$

The term $1/(\epsilon_\lambda+\epsilon_\mu)$ may be preceded by $1/\epsilon_\lambda$ or $1/\epsilon_\mu$, giving, for the two terms, $(1/\epsilon_\lambda+1/\epsilon_\mu)/(\epsilon_\lambda+\epsilon_\mu)=1/\epsilon_\lambda\epsilon_\mu$. It is readily seen that allowing for this change in order gives

$$(-)^{n_\lambda+n_\mu+\cdots.}/\{n_\lambda!\;\epsilon_\lambda^{n_\lambda}.n_\mu!\;\epsilon_\mu^{n_\mu}\ldots.\}.$$

Thus the emissions occurring before scattering give the partial matrix element

$$(2\pi e^2/\hbar c)^{(n_\lambda+n_\mu+\ldots.)/2}.(\bar{\epsilon}/\epsilon_\lambda)^{3n_\lambda/2}(\bar{\epsilon}/\epsilon_\mu)^{3n_\mu/2}\ldots.$$
$$\times[(\mathbf{p}_0.\mathbf{e}_\lambda)/mc]^{n_\lambda}[(\mathbf{p}_0.\mathbf{e}_\mu)/mc]^{n_\mu}\ldots.1/\sqrt{(n_\lambda!\;n_\mu!\ldots.)}$$

where $\bar{\epsilon}=\hbar c/L$ is the minimum photon energy. If there had been $\bar{n}_\lambda$, $\bar{n}_\mu$, ... etc. photons present initially, the factorial term would be replaced by

$$\frac{1}{n_\lambda!}\sqrt{\left(\frac{(n_\lambda+\bar{n}_\lambda)!}{\bar{n}_\lambda!}\right)}\frac{1}{n_\mu!}\sqrt{\left(\frac{(n_\mu+\bar{n}_\mu)!}{\bar{n}_\mu!}\right)}\ldots.$$

It is now possible to compare the various subtraction methods. Heitler and Peng (1942) suggest using the lowest order elements joining the initial and final states. Thus for $H_{\bar{n}_\lambda\bar{p},\,n_\lambda p}$, where (p,n_λ), $(\bar{p},\bar{n}_\lambda)$ are the initial and final states respectively, their method gives

$$(2\pi e^2/\hbar c)^{(\bar{n}_\lambda-n_\lambda)/2}(\bar{\epsilon}/\epsilon_\lambda)^{3(\bar{n}_\lambda-n_\lambda)/2}$$
$$\times\sum_{n'=n_\lambda}^{\bar{n}_\lambda}\frac{(\bar{\mathbf{p}}.\mathbf{e}_\lambda)^{\bar{n}_\lambda-n'}(\mathbf{p}.\mathbf{e}_\lambda)^{n'-n_\lambda}}{(\bar{n}_\lambda-n')!\,(n'-n_\lambda)!}(-)^{\bar{n}_\lambda-n'}.\sqrt{\left(\frac{\bar{n}_\lambda!}{n_\lambda!}\right)}V_{\bar{p}p},$$

assuming $\bar{n}_\lambda\geqslant n_\lambda$. Thus

$$H_{\bar{n}_\lambda\bar{p},\,n_\lambda p}=\frac{(\mathbf{a}_\lambda.\mathbf{p}-\bar{\mathbf{p}})^{\bar{n}_\lambda-n_\lambda}}{(\bar{n}_\lambda-n_\lambda)!}.\sqrt{\left(\frac{\bar{n}_\lambda!}{n_\lambda!}\right)}V_{\bar{p}p},\qquad\ldots\ldots(19\,a)$$

where $\mathbf{a}_\lambda=\mathbf{e}_\lambda(2\pi e^2/\hbar c)^{\frac{1}{2}}(\bar{\epsilon}/\epsilon_\lambda)^{\frac{3}{2}}$.

If $n_\lambda\geqslant\bar{n}_\lambda$, the result is

$$H_{\bar{n}_\lambda\bar{p},\,n_\lambda p}=\frac{(\mathbf{a}_\lambda.\bar{\mathbf{p}}-\mathbf{p})^{n_\lambda-\bar{n}_\lambda}}{(n_\lambda-\bar{n}_\lambda)!}.\sqrt{\left(\frac{n_\lambda!}{\bar{n}_\lambda!}\right)}V_{\bar{p}p},\qquad\ldots\ldots(19\,b)$$

$V_{\bar{p}p}$ could be replaced by the matrix element for the emission of a high frequency photon without destroying the argument. It will be seen that these matrix elements are just the first term in the expression (4), after omitting the exponential factor. The emission or absorption of other photons merely introduces multiplicative terms got by replacing λ in (19 *a*) and (19 *b*) by μ, ν, ... etc. For an electron such that $p, \bar{p} \ll mc$, it is readily seen that $(\mathbf{a}_\lambda \cdot \bar{\mathbf{p}} - \mathbf{p}) \ll 1$.

The most important property of the coefficients $(mp\,|\,np')$ of the Bloch–Nordsieck method is

$$\Sigma_n (mp\,|\,np')(np'\,|\,\bar{m}\bar{p}) = (mp\,|\,\bar{m}\bar{p}).$$

This relation makes it possible for the emission of low frequency photons to have a very small effect upon the electron's motion, as the low frequency part factorizes out of the damping equations (cf. § 4).

Consider the damping equation for the emission of one high frequency and no low frequency photons using (19). It will contain a term of the type $\Sigma_{n'} H_{0_\lambda \bar{p}j,\, n'_\lambda p'} U_{n'_\lambda p', 0_\lambda p}$. If the emission of low frequency photons is not to influence the scattering of the electron (beyond the small effect due to neglecting the emitted energy and momentum), this term should reduce to the form $H_{pj,\,p'} U_{p',p}$. From the classical result it is clear that $U_{1_\lambda p', 0_\lambda p}$ should contain the factor $(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}')$. Summing over n' up to 1 gives

$$H_{pj,\,p'} U_{p',p} + H_{pj,\,p'} U_{p',p} (\mathbf{a}_\lambda \cdot \bar{\mathbf{p}} - \mathbf{p}')(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}').$$

Thus there is a correcting factor $\{1 + (\mathbf{a}_\lambda \cdot \bar{\mathbf{p}} - \mathbf{p}')(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}')\}$. This factor does not allow $\mathbf{p}'$ to drop out of the expression, and it is hard to see how higher terms in n' can rectify that. Also, although for a single low frequency photon the factor is almost unity, when all low frequency photons are considered it becomes

$$\Pi_\lambda \{1 + (\mathbf{a}_\lambda \cdot \bar{\mathbf{p}} - \mathbf{p}')(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}')\} \simeq \exp\{\Sigma_\lambda (\mathbf{a}_\lambda \cdot \bar{\mathbf{p}} - \mathbf{p}')(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}')\},$$

which diverges as the lowest energy $\bar{\epsilon} \to 0$. There is thus a very strong damping, which will greatly influence the angular distribution of the electron wave function. Other forms of $U_{1_\lambda p', 0_\lambda p}$, including higher order terms in $(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}')$, may be tried, but none of these can cancel the term $(\mathbf{a}_\lambda \cdot \bar{\mathbf{p}})(\mathbf{a}_\lambda \cdot \mathbf{p}')$ occurring in $\{1 + (\mathbf{a}_\lambda \cdot \bar{\mathbf{p}} - \mathbf{p}')(\mathbf{a}_\lambda \cdot \mathbf{p} - \mathbf{p}')\}$ without integrating over $\mathbf{p}'$, which again introduces a strong damping of an admissible type into the electron's motion.

An alternative procedure of subtraction (Hamilton 1949 b) is to neglect all self energy terms in the original and virtual states. This means neglecting compound matrix elements in which any two states (intermediate or final) have precisely the same quantum numbers or descriptions. Such matrix elements are diagonal terms of the Hamiltonian (if the identical states are initial and final) or contain diagonal terms relating to some intermediate state.

The matrix elements derived in this fashion contain more terms than those of Heitler and Peng. For example, the element $H_{\bar{n}_\lambda \bar{p}, 0_\lambda p}$ (to take a simple case) includes terms allowing for the emission of n_μ photons ϵ_μ, n_ν photons ϵ_ν, ... etc. before the scattering by V, and the re-absorption of these photons after the scattering. This gives

$$H_{\bar{n}_\lambda \bar{p}, 0_\lambda p} = \prod_{\mu \neq \lambda} \exp\{(\mathbf{p} \cdot \mathbf{a}_\mu)(\bar{\mathbf{p}} \cdot \mathbf{a}_\mu)\} \cdot \left[\frac{1}{\sqrt{(\bar{n}_\lambda !)}} (\mathbf{p} \cdot \mathbf{a}_\lambda)^{\bar{n}_\lambda} \{\exp[(\mathbf{a}_\lambda \cdot \mathbf{p}_\lambda)(\mathbf{a}_\lambda \cdot \bar{\mathbf{p}}_\lambda)] - 1\} + \frac{1}{\sqrt{(\bar{n}_\lambda !)}} (\mathbf{a}_\lambda \cdot \mathbf{p} - \bar{\mathbf{p}})^{\bar{n}_\lambda} \right] \cdot V_{\bar{p}p}.$$

This formula actually bears more relation to $(\bar{n}_\lambda \bar{p} \,|\, 0p)$ of (4) than does the matrix element derived above.

However, there is another type of term to be added. Consider the process $(0_\lambda, \mathbf{p}) \rightarrow (1_\lambda, \bar{\mathbf{p}})$ during scattering by V. Then there is a term $(\mathbf{a}_\lambda \cdot \mathbf{p})$ arising from the emission of photon λ before the scattering. However, one can also have the process $\nu_e \lambda_e \nu_a$, where 'e' and 'a' mean emission and absorption respectively. This gives a term of the order $(\mathbf{a}_\lambda \cdot \mathbf{p})(\mathbf{a}_\nu \cdot \mathbf{p})^2$, and the only proviso is that ν and λ do not represent the same photon. Another allowed process is $\nu_e \mu_e \nu_a \lambda_e \mu_a$, giving a term of the order $(\mathbf{a}_\lambda \cdot \mathbf{p})(\mathbf{a}_\nu \cdot \mathbf{p})^2(\mathbf{a}_\mu \cdot \mathbf{p})^2$, where $\nu \neq \lambda$ and $\mu \neq \nu$ or λ. As a result the original term $(\mathbf{a}_\lambda \cdot \mathbf{p})$ is modified by a factor of the type

$$\prod_{\nu \neq \lambda} \{1 + \gamma_\nu (\mathbf{a}_\nu \cdot \mathbf{p})^2\} \cdot \prod_{\mu \neq \nu, \lambda} \{1 + \delta_\mu (\mathbf{a}_\mu \cdot \mathbf{p})^2\},$$

where γ_ν, δ_μ are factors of the order of magnitude of unity. Further processes of the type $\nu_e \mu_e \nu_a \tau_e \mu_a \lambda_e \tau_a$, ... etc. can occur, and as a result the product of a large number of factors, each of which are of the order of $\exp\{\Sigma_\nu (\mathbf{a}_\nu \cdot \mathbf{p})^2\}$, can enter the matrix element. The method of excluding self energy terms is therefore useless in the present problem. The reason for its failure lies in the strong interaction of the electron with low frequency virtual photons.

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Directional Correlation between Successive Internal-Conversion Electrons

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ABSTRACT. For the internal conversion of successive electric multipole quanta by s-electrons, unaccompanied by spin reversal, the directional correlation between the ejected electrons can be derived by symmetry arguments alone. A general formula is established and special cases likely to be of practical interest are tabulated. Comparison is made with Hamilton's results for γ–γ correlation.

§ 1. INTRODUCTION

When an atomic nucleus passes from an excited level A to the ground level C via a second excited level B, by the emission of two successive particles or quanta, one expects from general considerations that the direction of the second emission will be related to that of the first, provided the lifetime of the level B is small compared with the precession period of the nucleus in whatever external field it finds itself. This correlation is expected to depend on the angular momenta of the nuclear levels A, B and C and the angular momenta of the emitted particles or quanta.

One of the earliest calculations of angular correlation in processes of the above type was that by Hamilton (1940), who worked out the correlation between successive γ-quanta emitted by a nucleus in the absence of an external field. Goertzel (1946) extended this work to include the effect of external fields, with particular reference to the magnetic field of the extra-nuclear electrons. More recently there have appeared other calculations, based on various types of interaction, of angular correlation in processes involving β- and γ-decay. References to, and discussion of, these are given by Yang (1948) in a general paper on angular correlation in nuclear processes.

Yang shows that the general form of the angular distribution of the products of nuclear reactions and disintegrations can be derived from symmetry considerations alone, regardless of the particular form of interaction at work. The correlation between successive particles or quanta is specified by the probability $I(\theta)$ that the direction of propagation of the second makes an angle θ with that of the first. Using only the invariance of the properties of the physical process under space rotation and under inversion, Yang is able to expand $I(\theta)$ as a power series in $\cos\theta$, in which the highest power of $\cos\theta$ is severely limited by the angular momenta of the nuclear levels and the angular momenta of the emitted particles or quanta, so that the expansion frequently reduces to a very few terms. The processes which he considers include β–neutrino, β–γ and γ–γ correlations; also the general nuclear reaction $M+P\rightarrow N+Q$, θ in this case being the angle between the propagation directions of the incident particle P and the emergent particle Q, and M and N being the initial and final nuclei.*

* Myers (1938) has given a detailed calculation, based on symmetry arguments, of the angular distribution of resonance disintegration products. This calculation is not as general as Yang's but within the limits of its validity it gives quantitative results, whereas Yang's gives information only about the general form of the distribution.

In general, to evaluate the ratios of the coefficients of the various powers of $\cos\theta$ appearing in the expansion of $I(\theta)$ one must make specific assumptions about the interaction. There are, however, certain types of disintegration in which the number of variables is sufficiently small to permit calculation of these ratios merely by an extension of the symmetry arguments. Such a disintegration is one in which the nuclear transitions A→B→C are achieved by the ejection from the atom of two successive internal-conversion electrons, subject to certain restrictions stated in the next section. It is the purpose of the present paper to consider such a disintegration by symmetry methods, without specifying the mechanism of internal conversion, and in particular leaving aside the question of whether or not the ejection of an orbital electron is pictured as due to the nucleus emitting and the electron absorbing a γ-quantum. (In the relativistic treatment of internal conversion (cf. Mott and Taylor 1932, 1933) no sharp distinction can be made between the effect of direct interaction and the effect of γ-ray emission and subsequent absorption.)

The function $I(\theta)$ expressing the angular correlation between the emission directions of successive conversion electrons is expanded in a series of even spherical harmonics whose coefficients are explicit functions of the angular momenta of the nuclear levels and the emitted electrons. These coefficients are tabulated and their general behaviour discussed for most cases likely to be of practical interest. The results are compared with those of Hamilton for γ–γ correlation. In conclusion we discuss the extent to which the initial simplifying assumptions limit the applicability of the results.

At least one report of observed angular correlation between successive conversion electrons has appeared (Ward and Walker 1949); β–neutrino and γ–γ correlations have been studied experimentally by many workers, references being given in Yang's paper.

§ 2. DERIVATION OF GENERAL CORRELATION FORMULA

We denote the total angular momenta of the nuclear levels A, B and C by the vectors $\mathbf{J}_A, \mathbf{J}_B, \mathbf{J}_C$ and the orbital angular momenta of the ejected electrons by $\mathbf{l}_1, \mathbf{l}_2$; thus in units of $\hbar$, J_A, J_B and J_C may be integer or half-integer, l_1 and l_2 only integer. The projections of these vectors on the (arbitrary) z-axis we denote by M_A, M_B, M_C, m_1, m_2. It is assumed that: (*a*) the total angular momentum of all orbital electrons, other than the ejected ones, remains constant throughout; (*b*) the ejected electrons have come from an s-state (but not necessarily the K-shell); (*c*) internal conversion occurs without spin reversal of the ejected electrons.

These assumptions have the effect of restricting the present calculations to internal conversion of electric multipole radiation only (cf. § 5).

Conservation of angular momentum, together with the above assumptions, gives immediately the following vector equations:

$$\mathbf{J}_A = \mathbf{J}_B + \mathbf{l}_1, \qquad \ldots\ldots (1)$$

$$\mathbf{J}_B = \mathbf{J}_C + \mathbf{l}_2, \qquad \ldots\ldots (2)$$

with components in the z-direction:

$$M_A = M_B + m_1, \qquad \ldots\ldots (3)$$

$$M_B = M_C + m_2. \qquad \ldots\ldots (4)$$

In the final state, the whole system, consisting of the nucleus plus the two ejected electrons,* is still characterized by the total angular momentum $\mathbf{J}_A$ with component M_A; we therefore introduce $\psi_A(J_A, M_A)$ for the wave function of the whole system in the final state. Similarly the system consisting of the nucleus plus the second ejected electron is still characterized in the final state by $\mathbf{J}_B$, M_B, so for the wave function describing it we introduce $\psi_B(J_B, M_B)$, and $\psi_C(J_C, M_C)$ for the nucleus by itself in the final state. The wave functions of the ejected electrons by themselves we denote by $\psi_1(l_1, m_1)$, $\psi_2(l_2, m_2)$. Where no ambiguity occurs we write $\psi_A(J_A, M_A)$ simply as ψ_A, with similar abbreviations for the other wave functions.

Because of the vector relation (1), ψ_A is expressible as a linear combination of the $2J_B+1$ possible ψ_B corresponding to a definite J_B, and the $2l_1+1$ possible ψ_1 corresponding to a definite l_1 (see for example Condon and Shortley 1935). Thus

$$\psi_A \sim \sum_{M_B,\, m_1} C^{J_A M_A}_{J_B M_B l_1 m_1} \psi_B \psi_1 . \qquad \ldots\ldots (5)$$

Similarly, because of (2), ψ_B may be written as a linear combination of the $2J_C+1$ possible ψ_C and the $2l_2+1$ possible ψ_2, which may then be substituted in (5), giving:

$$\psi_A \sim \sum_{M_B,\, m_1,\, M_C,\, m_2} C^{J_A M_A}_{J_B M_B l_1 m} \, C^{J_B M_B}_{J_C M_C l_2 m_2} \psi_C \psi_1 \psi_2 . \qquad \ldots\ldots (6)$$

The C's in (5) and (6) are the coefficients common to all problems in which two angular momenta are combined to give a resultant; equivalent general formulae for them have been given by Wigner (1931) and Van der Waerden (1932) using group theoretical methods. Racah (1942) has also given an elementary algebraic derivation of a formula which, apart from a normalizing factor, is the same as Van der Waerden's. In the present calculations it was found most convenient to use the formula of Racah, to whose paper, also, the reader is referred for a concise account of the orthogonality, symmetry and other useful properties of the C's.†

The angular parts of ψ_1, ψ_2 are the spherical harmonics $Y_{l_1, m_1}(\theta_1, \phi_1)$, $Y_{l_2, m_2}(\theta_2, \phi_2)$. Without loss of generality we may define our z-axis as the direction of emission of one (say the first) of the conversion electrons, by setting $\theta_1 = \phi_1 = 0$. Since $Y_{l_1 m_1}(0, 0)$ vanishes except for $m_1 = 0$, (3) and (4) reduce to

$$M_A = M_B = M_C + m_2 . \qquad \ldots\ldots (7)$$

Thus the sum over M_B, m_1 in (6) reduces to one term. Moreover, θ_2, ϕ_2 now define the emission direction of the second electron relative to that of the first, so that the probability $I(\theta, \phi)\, d\Omega$ that the second electron is emitted into the solid angle $d\Omega$ about the direction (θ, ϕ)—the suffix 2 may now be dropped—expresses directly the angular correlation between the electrons. This probability function is obtained from (6) by

$$I(\theta, \phi) \sim \sum_{M_A} \int \psi_A^* \psi_A \, d\tau , \qquad \ldots\ldots (8)$$

* In view of assumption (*a*) the other orbital electrons can be omitted from the discussion without affecting the validity of the symmetry arguments to be used.

† Our notation differs from Racah's: we define $C^{jm}_{j_1 m_1 j_2 m_2}$ to be the quantity which he writes $(j_1 j_2 m_1 m_2 / j_1 j_2 j m)$.

where the integration is over all coordinates except θ, ϕ, and where the summation over M_A has to be introduced because $\psi_A(J_A, M_A)$ refers to a definite M_A, whereas experimentally it is the average effect over all the $2J_A+1$ possible values of M_A that is observed. The integrand in (8) is a sum, over all possible M_C and m_2 consistent with (7), of such terms as

$$\{C^{J_A M_A}_{J_B M_A l_1 0}\, C^{J_B M_A}_{J_C M_C l_2 m_2} \psi_C(J_C, M_C) Y_{l_2, m_2}(\theta, \phi)\}^* \times \{C^{J_A M_A}_{J_B M_A l_1 0}\, C^{J_B M_A}_{J_C M_C' l_2 m_2'} \psi_C(J_C, M_C') Y_{l_2, m_2'}(\theta, \phi)\}. \quad \ldots\ldots (9)$$

(The constant factor $Y_{l_1}0(0,0)$ has been omitted; other constant factors will be omitted from time to time without special mention.) Because of the orthogonality of the ψ_C the only terms which remain after integration are those in which $M_C' = M_C$, and hence $m_2' = m_2$. Noting the reality of the C's, and that $|Y_{l_2, m_2}(\theta,\phi)|^2$ is independent of ϕ, we see that our angular distribution formula becomes

$$I(\theta) \sim \sum_{M_A, M_C, m_2} \{C^{J_A M_A}_{J_B M_A l_1 0}\, C^{J_B M_A}_{J_C M_C l_2 m_2} Y_{l_2, m_2}(\theta, 0)\}^2. \quad \ldots\ldots (10)$$

The fact that this expression is independent of ϕ means that the distribution of the second electron is symmetrical about the direction of the first, while the fact that only squares of spherical harmonics appear means that the distribution is symmetrical about the equatorial plane.

As an example of the use of (10) we may consider the case in which $J_A = J_C = 0$, and J_B has any integral value, say L. We then have immediately from (1)–(4) $l_1 = l_2 = L$, $M_A = M_C = m_2 = 0$, so that the sum in (10) reduces to one term, and $I(\theta) \sim \{Y_{L,0}(\theta, 0)\}^2$. In particular when $L=1$, $I(\theta) \sim \cos^2\theta$).

However, apart from such extremely simple cases, (10) as it stan ls is of little value for practical calculation, for if the sum includes more than a very few terms the labour of evaluating it directly becomes prohibitive. It will now be shown that, using an expansion of $\{Y_{l_2, m_2}(\theta, 0)\}^2$ linear in spherical harmonics, together with Racah's results (1942) for summations of products of C's, (10) can be manipulated into a form more convenient both for practical calculation and for making general inferences about the nature of the angular distribution.

§ 3. ANALYSIS AND DISCUSSION OF GENERAL FORMULA

By the customary Fourier method we may obtain the expansion

$$\{Y_{l_2, m_2}(\theta, 0)\}^2 = \sum_k A_k Y_{k,0}(\theta, 0), \quad \ldots\ldots (11)$$

where the coefficients are found to be given by

$$A_k = \{(-1)^{m_2}(2l_2+1)/2\pi^{\frac{1}{2}}(2k+1)^{\frac{1}{2}}\}\, C^{k0}_{l_2 0 l_2 0}\, C^{k0}_{l_2 - m_2 l_2 m_2}. \quad \ldots\ldots (12)$$

Substitution of (11) and (12) in (10) gives $I(\theta)$ as a linear expansion in spherical harmonics:

$$I(\theta) \sim \sum_k B_k Y_{k,0}(\theta, 0) \quad \ldots\ldots (13)$$

with

$$B_k = (2k+1)^{-\frac{1}{2}}\, C^{k0}_{l_2 0 l_2 0} \sum_{M_A, M_C, m_2} (-1)^{m_2} \{C^{J_A M_A}_{J_B M_A l_1 0}\, C^{J_B M_A}_{J_C M_C l_2 m_2}\}^2\, C^{k0}_{l_2 - m_2 l_2 m_2}. \quad \ldots\ldots (14)$$

We see that B_k involves the summation, over certain common arguments, of the product of five C's. Now Racah gives (in effect) the following result for the sum of products of three C's:

$$\sum_{\alpha,\beta,\delta} C^{c\gamma}_{a\alpha b\beta}\, C^{e\epsilon}_{d\delta b\beta}\, C^{d\delta}_{a\alpha f\phi} = (2c+1)^{\frac{1}{2}}(2d+1)^{\frac{1}{2}} W(cbfd;\ ae)\, C^{e\epsilon}_{c\gamma f\phi} \quad \ldots\ldots (15)$$

The general formula for the function W and an account of its properties appear in his paper (1942). (Like the C's, the W's have useful symmetry and orthogonality properties.) It will be shown that a double application of (15) enables the sum in (14) to be evaluated. First let us consider the summation over all possible values of M_C, m_2 subject to $M_C + m_2 = M_A$, whilst M_A is kept constant. We have then to evaluate

$$\sum_{M_C, m_2} (-1)^{m_2} \{C^{J_B M_A}_{J_C M_C l_2 m_2}\}^2\, C^{k0}_{l_2 - m_2 l_2 m_2}.$$

Using the symmetry properties of the C's, it is readily shown that, apart from factors independent of M_C and m_2, this sum may be expressed as

$$\sum_{M_C, m_2} \{C^{J_B M_A}_{l_2 m_2 J_C M_C}\}^2\, C^{l_2 m_2}_{l_2 m_2 k0},$$

which is a form directly evaluable from (15) on making the substitutions

$c, e \to J_B$	$a, d \to l_2$	$b \to J_C$	$f \to k$
$\gamma, \epsilon \to M_A$	$\alpha, \delta \to m_2$	$\beta \to M_C$	$\phi \to 0$

After carrying out these substitutions and inserting the factors independent of M_C and m_2, we have the result:

$$\sum_{M_C, m_2} (-1)^{m_2} \{C^{J_B M_A}_{J_C M_C l_2 m_2}\}^2\, C^{k0}_{l_2 - m_2 l_2 m_2}$$
$$= (2k+1)^{\frac{1}{2}}(2J_B+1)^{\frac{1}{2}}(-1)^{k-l_2} W(J_B J_C k l_2;\ l_2 J_B)\, C^{J_B M_A}_{J_B M_A k0}. \quad \ldots\ldots (16)$$

Substituting (16) in (14), we see that the dependence on M_A is now limited to the factors $\{C^{J_A M_A}_{J_B M_A l_1 0}\}^2$, $C^{J_B M_A}_{J_B M_A k0}$, so that the summation over M_A reduces effectively to summing this product of three C's over all M_A from $-J_A$ to J_A, and this can be done as above using (15) again. (Actually a slightly different formula, obtained from (15) by the symmetry properties of the W's, is more suitable in this case.) The result of this summation is

$$\sum_{M_A} \{C^{J_A M_A}_{J_B M_A l_1 0}\}^2\, C^{J_B M_A}_{J_B M_A k0}$$
$$= (2J_A+1)(2J_B+1)^{\frac{1}{2}}(2k+1)^{-\frac{1}{2}}(-1)^{-l_1} W(J_B J_A k l_1;\ l_1 J_B)\, C^{k0}_{l_1 0 l_1 0}. \quad \ldots\ldots (17)$$

Thus (16) and (17) in conjunction with (14) enable B_k to be expressed as a product of 2 W's, 2 C's and a few odd factors. Factors independent of k may, of course, be dropped on substituting B_k in (13); the factor $(2k+1)^{-\frac{1}{2}}$ may also be removed if we elect to expand $I(\theta)$ in terms of $P_k(\cos\theta)$ rather than $Y_{k,0}(\theta, 0)$. [$P_k(\cos\theta)$ is the Legendre polynomial normalized so that $P_k(1) = 1$, and $Y_{k,0}(\theta, 0)$ is by definition $\{(2k+1)/4\pi\}^{\frac{1}{2}} P_k(\cos\theta)$.] The result then assumes the rather neat form

$$I(\theta) \sim \sum_k C^{k0}_{l_1 0 l_1 0}\, C^{k0}_{l_2 0 l_2 0} W(J_B J_A k l_1;\ l_1 J_B) W(J_B J_C k l_2;\ l_2 J_B) P_k(\cos\theta). \quad \ldots\ldots (18)$$

If in (18) we interchange J_A with J_C, and l_1 with l_2, we see that $I(\theta)$ is unaffected, in accordance with the physical consideration that it expresses merely the relative angular distribution of the electrons. The properties of either the C's or the W's limit k in (18) to the values 0, 2, 4 .... up to $2l_1$ or $2l_2$, whichever is the lower, showing that $I(\theta)$ is symmetrical about the equatorial plane. The latter result we had already deduced from the appearance of $\{Y_{l_2, m_2}(\theta),0\}^2$ in formula (10) for $I(\theta)$; the limitation on the highest value of k is in accordance with the consideration that we could have taken the z-axis in the direction of the second rather than the first electron and obtained a formula physically equivalent to (10) but expressing $I(\theta)$ in terms of $\{Y_{l_1 m_1}(\theta, 0)\}^2$. Moreover (18) reveals a further limitation on k which is not apparent from (10), for the W functions vanish unless $k \leqslant 2J_B$. Thus $J_B = 0$ or $\frac{1}{2}$ gives an isotropic distribution, whatever l_1 and l_2.

The limitations on $I(\theta)$ discussed in the preceding paragraph apply also to the directional correlation of successive γ-quanta, l_1 and l_2 in this case being interpreted as the multipole orders of the two quanta (Hamilton 1940, Goertzel 1946).

§4. PRACTICAL APPLICATION OF GENERAL FORMULA

(i) *Tabulation of the W Functions*

In the practical application of (18) numerical values of the C's and W's must be obtained either from tables or by direct calculation. Tables of the C's for values of the arguments likely to arise in practical cases have been published (see for example Condon and Shortley 1935); however, for C's of the type C^{c0}_{a0b0} appearing in (18) the summation formula for the C with general arguments reduces to a closed form quite suitable for computation.

A series of tables for $W(abcd; ef)$ may be drawn up, each one characterized by a definite value: 0, $\frac{1}{2}$, 1, $\frac{3}{2}$ .... of one of the arguments, say d. In each table the other arguments are kept general, i.e. are allowed to assume any values consistent with the limitations detailed by Racah (1942, p. 444). For a definite numerical value, say j, of d these limitations restrict f to the values $b+j$, ..., $|b-j|$ and e to the values $c+j$, ..., $|c-j|$. The entries in each table are arranged according to these definite values of f and e; there will thus be in general $(2j+1)^2$ entries in the table for $d=j$, although when b, c have numerical values less than j some of these entries will be inadmissible.

The selection of d as the argument to characterize each table by a definite numerical value was quite arbitrary, any other argument being equally suitable; any W having one or more of its arguments equal to the numerical value assigned to d in a particular table can be easily obtained from that table by use of the symmetry properties of the W's. So far as the author is aware no published tables of W functions exist*, but their compilation for the lowest numerical values of d is not too laborious; as d increases, however, not only does the number of entries in each table increase as the square of the natural numbers, but the entries themselves become more and more complicated.

(ii) *Simplification when l_1 and l_2 have Lowest Allowed Values*

It is known that for a given nuclear transition accompanied by the ejection of an internal-conversion electron of wave number k_e the probability that the ejected electron has angular momentum l varies in general as $(k_e r)^2$, r being the

* Tables for $d \leqslant 2$ have been prepared by Professor H. A. Jahn of Southampton for publication in due course, in connection with work on nuclear energy levels.

nuclear radius. Where this is true—exceptions are discussed in the next paragraph—only the lowest value of l permitted by the magnitude of ΔJ and by the parity selection rules need be considered: higher values of l may occur but only with insignificant probability, since in all practical cases $k_e r \leqslant 1$. (Only for energies greater than 20 MEV. would $k_e r$ approach unity.)

For the majority of nuclei, owing to the approximate coincidence of the centres of charge and mass, the transition probabilities for $l=1$ and $l=2$ are comparable, and not in the ratio $(k_e r)^2 : (k_e r)^4$. However, even if ΔJ permits both $l=1$ and $l=2$, one of these alternatives will be eliminated by the parity rules, and since the transition probabilities for higher l fall off according to the $(k_e r)^{2l}$ rule, the argument for retaining only the lowest allowed l is not vitiated. More special nuclear symmetries may on occasion make it necessary to retain higher values of l.

Excepting such special cases, the practical applicability of (18) will not be reduced if we limit l_1 to either of the values $|J_A - J_B|$ or $|J_A - J_B|+1$, whichever is allowed by the parity relations in a particular case, and similarly limit l_2 to $|J_B - J_C|$ or $|J_B - J_C|+1$. The general summation formulae for $W(J_B J_A k l_1;\ l_1 J_B)$ and $W(J_B J_C k l_2;\ l_2 J_B)$ reduce to one or two terms on substituting these values of l_1 and l_2 respectively, and a good deal of cancellation occurs on working out the coefficient of $P_k(\cos\theta)$ in (18). If we write $2g$ for k, so that g assumes successive integer values, it is found that the angular distribution function can now be written

$$I(\theta) \sim \sum_g b_{2g}\{[4g+1][(2g)!]^2/[g!]^4\} P_{2g}(\cos\theta). \qquad \ldots\ldots (19)$$

The coefficients b_{2g} are functions only of g and the three J's; there are sixteen of them corresponding to the various possible combinations of $l_1 = |J_A - J_B|$ or $|J_A - J_B|+1$ with $l_2 = |J_B - J_C|$ or $|J_B - J_C|+1$ for $J_A \gtrless J_B$ and $J_B \gtrless J_C$. In tabulating these coefficients the substitutions $J_B \to J$, $J_A - J_B \to \Delta$, $J_C - J_B \to \delta$ shorten the formulae somewhat. It will be observed in Table 1 that only ten of the sixteen formulae are given explicitly, the remainder being readily obtained from these ten by symmetry relations.

[*continued overleaf*

Table 1. Coefficients b_{2g} in the Angular Distribution

$$I(\theta)\sim\sum_{g} b_{2g}\{[4g+1][(2g)!]^2/[g!]^4\}P_{2g}(\cos\theta)$$

for the Nuclear Transition $J+\Delta\to J\to J+\delta$ with Ejection of Electrons with Angular Momenta l_1, l_2

Δ	δ	b_{2g}
$-l_1$	$-l_2$	$\dfrac{(g-\Delta)!\,(2J-2g)!\,(2J+2g+1)!\,(g-\delta)!}{(2g+1-2\Delta)!\,(-\Delta-g)!\,(2g+1-2\delta)!\,(-\delta-g)!}$
$-l_1$	$-l_2+1$	$\dfrac{(g-\Delta)!\,(2J+2g+1)!\,(2J-2g)!\,(g+1-\delta)!\,\{2(1-\delta)(J+1)-2g(2g+1)(J+\delta)\}}{(-\Delta-g)!\,(2g+1-2\Delta)!\,(1-g-\delta)!\,(2g+3-2\delta)!}$
$-l_1$	l_2-1	$\dfrac{(g-\Delta)!\,(g+\delta+1)!\,\{2(\delta+1)J-2g(2g+1)(J+\delta+1)\}}{(2g+1-2\Delta)!\,(-g-\Delta)!\,(2g+2\delta+3)!\,(\delta-g+1)!}$
$-l_1$	l_2	$\dfrac{(g-\Delta)!\,(g+\delta)!}{(2g+1-2\Delta)!\,(-g-\Delta)!\,(\delta-g)!\,(2\delta+2g+1)!}$
$-l_1+1$	$-l_2$	Interchange Δ and δ in $(-l_1,\ -l_2+1)$
$-l_1+1$	$-l_2+1$	$\dfrac{(g+1-\Delta)!\,(2J-2g)!\,(2J+2g+1)!\,(g+1-\delta)!\,\{2(1-\Delta)(J+1)-2g(2g+1)(J+\Delta)\}\{2(1-\delta)(J+1)-2g(2g+1)(J+\delta)\}}{(2g+3-2\Delta)!\,(1-g-\Delta)!\,(1-g-\delta)!\,(2g+3-2\delta)!}$
$-l_1+1$	l_2-1	$\dfrac{(g+1-\Delta)!\,(g+1+\delta)!\,\{2(1-\Delta)(J+1)-2g(2g+1)(J+\Delta)\}\times\{2(1+\delta)J-2g(2g+1)(J+\delta+1)\}}{(2g+3-2\Delta)!\,(1-g-\Delta)!\,(\delta+1-g)!\,(2\delta+2g+3)!}$
$-l_1+1$	l_2	$\dfrac{(g+1-\Delta)!\,(\delta+g)!\,\{2(1-\Delta)(J+1)-2g(2g+1)(J+\Delta)\}}{(2g+3-2\Delta)!\,(1-g-\Delta)!\,(\delta-g)!\,(2g+1+2\delta)!}$
l_1-1	$-l_2$	Interchange Δ and δ in $(-l_1,\ l_2-1)$
l_1-1	$-l_2+1$	Interchange Δ and δ in $(-l_1+1,\ l_2-1)$
l_1-1	l_2-1	$\dfrac{(g+\Delta+1)!\,(g+\delta+1)!\,\{2(\Delta+1)J-2g(2g+1)(J+\Delta+1)\}\times\{2(\delta+1)J-2g(2g+1)(J+\delta+1)\}}{(2g+2\Delta+3)!\,(\Delta-g+1)!\,(2J-2g)!\,(2J+2g+1)!\times(\delta-g+1)!\,(2g+2\delta+3)!}$
l_1-1	l_2	$\dfrac{(\delta+g)!\,(\Delta+g+1)!\,\{2(\Delta+1)J-2g(2g+1)(J+\Delta+1)\}}{(2\delta+2g+1)!\,(\delta-g)!\,(2J-2g)!\,(2J+2g+1)!\times(\Delta-g+1)!\,(2\Delta+2g+3)!}$
l_1	$-l_2$	Interchange Δ and δ in $(-l_1,\ l_2)$
l_1	$-l_2+1$	Interchange Δ and δ in $(-l_1+1,\ l_2)$
l_1	l_2-1	Interchange Δ and δ in $(l_1-1,\ l_2)$
l_1	l_2	$\dfrac{(\Delta+g)!\,(\delta+g)!}{(2\Delta+2g+1)!\,(\Delta-g)!\,(2J-2g)!\,(2J+2g+1)!\,(\delta-g)!\,(2\delta+2g+1)!}$

Table 2. Coefficients a_2 in the Angular Distribution $1+a_2P_2(\cos\theta)$ when $l_1=l_2=1$

Δ	δ	$a_2\times 5$	Δ	δ	$a_2\times 5$
-1	-1	$\frac{(2J+3)(J+1)}{J(2J-1)}$	0	1	$-\frac{(2J-1)}{(J+1)}$
-1	0	$-\frac{(2J+3)}{J}$	1	-1	Same as $(-1, 1)$
-1	1	1	1	0	Same as (0, 1)
0	-1	Same as $(-1, 0)$	1	1	$\frac{J(2J-1)}{(J+1)(2J+3)}$
0	0	$\frac{(2J-1)(2J+3)}{J(J+1)}$			

Table 3. Coefficients a_2 in the Angular Distribution $1+a_2P_2(\cos\theta)$ when $l_1=2$, $l_2=1$

Δ	δ	$a_2\times 7$	Δ	δ	$a_2\times 7$
-2	-1	$\frac{2(J+1)(2J+3)}{J(2J-1)}$	1	-1	Same as $(-1, 1)$
-2	0	$\frac{-2(2J+3)}{J}$	1	0	$\frac{(2J-1)(J+6)}{J(J+1)}$
-2	1	2	1	1	$\frac{-(2J-1)(J+6)}{(J+1)(2J+3)}$
-1	-1	$\frac{-(2J+3)(J-5)}{J(2J-1)}$	2	-1	Same as $(-2, 1)$
-1	0	$\frac{(2J+3)(J-5)}{J(J+1)}$	2	0	$\frac{-2(2J-1)}{(J+1)}$
-1	1	$\frac{-(J+6)}{J}$	2	1	$\frac{2J(2J-1)}{(J+1)(2J+3)}$

Table 4(*a*). Coefficients a_2 in the Angular Distribution $1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$ when $l_1=l_2=2$

Δ	δ	$a_2\times 49$	Δ	δ	$a_2\times 49$
-2	-2	$\frac{20(2J+3)(J+1)}{J(2J-1)}$	1	-2	Same as $(-2, 1)$
			1	-1	Same as $(-1, 1)$
-2	-1	$\frac{10(J-5)(2J+3)}{J(2J-1)}$	1	1	$\frac{5(J+6)^2(2J-1)}{J(2J+3)(J+1)}$
-2	1	$\frac{-10(J+6)}{J}$	1	2	$\frac{-10(J+6)(2J-1)}{(2J+3)(J+1)}$
-2	2	20	2	-2	Same as $(-2, 2)$
-1	-2	Same as $(-2, -1)$	2	-1	Same as $(-1, 2)$
-1	-1	$\frac{5(2J+3)(J-5)^2}{J(2J-1)(J+1)}$	2	1	Same as (1, 2)
-1	1	$\frac{5(J-5)(J+6)}{J(J+1)}$	2	2	$\frac{20J(2J-1)}{(2J+3)(J+1)}$
-1	2	$\frac{-10(J-5)}{(J-1)}$			

Table 4 (*b*). Coefficients a_4 in the Angular Distribution $1+a_2P_2(\cos\theta)+P_4(\cos\theta)$ when $l_1=l_2=2$

Δ	δ	$a_4\times 49$	Δ	δ	$a_4\times 49$
-2	-2	$\frac{(2J+5)(2J+3)(J+2)(J+1)}{(2J-1)(2J-3)J(J-1)}$	1	-2	Same as $(-2, 1)$
			1	-1	Same as $(-1, 1)$
-2	-1	$\frac{-2(2J+5)(2J+3)(J+2)}{J(2J-1)(J-1)}$	1	1	$\frac{4(2J+5)(2J-1)(J-1)(2J-3)}{(2J+3)(J+2)(J-1)J}$
-2	1	$\frac{-2(2J+5)}{J}$	1	2	$\frac{-2(2J-1)(2J-3)(J-1)}{(2J+3)(J+2)(J+1)}$
-2	2	1	2	-2	Same as $(-2, 2)$
-1	-2	Same as $(-2, -1)$	2	-1	Same as $(-1, 2)$
-1	-1	$\frac{4(2J+5)(J+2)(2J+3)(2J-3)}{J(2J-1)(J-1)(J+1)}$	2	1	Same as (1, 2)
-1	1	$\frac{4(2J-3)(2J+5)}{J(J+1)}$	2	2	$\frac{J(2J-1)(2J-3)(J-1)}{(2J+5)(2J+3)(J+2)(J+1)}$
-1	2	$\frac{-2(2J-3)}{(J+1)}$			

Table 5. Coefficients a_2 in the Angular Distribution $1+a_2P_2(\cos\theta)$ when $l_1=3$, $l_2=1$

Δ	δ	a_2	Δ	δ	a_2
-3	-1	$\frac{(2J+3)(J+1)}{3J(2J-1)}$	2	-1	$-\frac{1}{J}$
-3	0	$-\frac{(2J+3)}{3J}$	2	0	$\frac{2J-1}{J(J+1)}$
-3	1	$\frac{1}{3}$	2	1	$\frac{-(2J-1)}{(J+1)(2J+3)}$
-2	-1	$\frac{(2J-3)}{J(2J-1)}$	3	-1	Same as $(-3, 1)$
-2	0	$-\frac{(2J+3)}{J(J+1)}$	3	0	$\frac{-(2J-1)}{3(J+1)}$
-2	1	$\frac{1}{J+1}$	3	1	$\frac{1}{3}\frac{J(2J-1)}{(J+1)(2J+3)}$

Table 6 (*a*). Coefficients a_2 in the Angular Distribution $1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$, when $l_1=3$, $l_2=2$

Δ	δ	$a_2\times 7$	Δ	δ	$a_2\times 7$
-3	-2	$\frac{10(2J+3)(J+1)}{3J(2J-1)}$	2	-2	$\frac{-10}{J}$
-3	-1	$\frac{-5(2J+3)(J-5)}{3J(2J-1)}$	2	-1	$\frac{5(J-5)}{J(J+1)}$
-3	1	$\frac{-5(J+6)}{3J}$	2	1	$\frac{5(J+6)(2J-1)}{2J(J+1)(2J+3)}$
-3	2	$\frac{10}{3}$	2	2	$\frac{-10(2J-1)}{(2J+3)(J+1)}$
-2	-2	$\frac{10(2J+3)}{J(2J-1)}$	3	-2	Same as $(-3, 2)$
-2	-1	$\frac{-5(J-5)(2J+5)}{J(J+1)(2J-1)}$	3	-1	$\frac{-5(J-5)}{3(J+1)}$
-2	1	$\frac{-5(J+6)}{J(J+1)}$	3	1	$\frac{-5(J+6)(2J-1)}{3(2J+3)(J+1)}$
-2	2	$\frac{10}{(J+1)}$	3	2	$\frac{10J(2J-1)}{3(2J+3)(J+1)}$

Table 6 (*b*). Coefficients a_4 in the Angular Distribution $1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$ when $l_1=3$, $l_2=2$

Δ	δ	$a_4\times 77$	Δ	δ	$a_4\times 77$
-3	-2	$\frac{3(2J+5)(J+2)(2J+3)(J+1)}{J(2J-1)(J-1)(2J-3)}$	2	-2	$\frac{-(7J+30)}{J}$
-3	-1	$\frac{-6(2J+5)(J+2)(2J+3)}{J(2J-1)(J-1)}$	2	-1	$\frac{2(2J-3)(7J+30)}{J(J+1)}$
-3	1	$\frac{-6(2J+5)}{J}$	2	1	$\frac{(7J+30)(2J-1)(J-1)(2J-3)}{J(J+2)(2J+3)(J+1)}$
-3	2	3	2	2	$\frac{-(7J+30)(2J-1)(J-1)(2J-3)}{(2J+5)(J+2)(2J+3)(J+1)}$
-2	-2	$\frac{-(7J-23)(2J+5)(J+2)(2J+3)}{J(2J-1)(J-1)(2J-3)}$	3	-2	Same as $(-3, 2)$
-2	-1	$\frac{2(7J-23)(2J+5)(J+2)(2J+3)}{J(J+1)(2J-1)(J-1)}$	3	-1	$\frac{-6(2J-3)}{(J+1)}$
-2	1	$\frac{2(7J-23)(2J+5)}{J(J+1)}$	3	1	$\frac{-6(2J-1)(J-1)(2J-3)}{(J+2)(2J+3)(J+1)}$
-2	2	$\frac{-(7J-23)}{(J+1)}$	3	2	$\frac{3J(2J-1)(J-1)(2J-3)}{(2J+5)(J+2)(2J+3)(J+1)}$

(iii) *Coefficients of* $P_2(\cos\theta)$ *and* $P_4(\cos\theta)$ *for definite numerical* l_1 *and* l_2

Tables 2–6 have been obtained from Table 1 by substituting the numerical values $l_1=1, 2, 3$; $l_2=1, 2$. If in Tables 3, 5, 6 (*a*) and 6 (*b*) we merely interchange the column headings Δ and δ they become respectively the tables for $l_1=1$, $l_2=2$; $l_1=1$, $l_2=3$; $l_1=2$, $l_2=3$, a result which follows from the symmetry of the general formula (18). It will be recalled that $I(\theta)$ is always isotropic for $J=0$ or $\frac{1}{2}$; Tables 1–6 are therefore applicable only for $J\geqslant 1$. It will be clear moreover that certain entries are applicable only for $J\geqslant 2$ and others only for $J\geqslant 3$, owing to limitations imposed by the angular momentum relations (1) and (2).

Tables 2–6 refer to angular distributions which contain Legendre polynomials up to the fourth order. If required, more complicated distributions may be calculated in exactly the same way by substituting the appropriate values of l_1, l_2 in Table 1.

In Figure 1 the coefficients a_2, for the case $l_1=l_2=1$ (Table 2), are plotted against J. It is seen that for increasing J each a_2 tends monotonically to some non-vanishing limit, say $(a_2)_\infty$, in the range $-\frac{2}{5}\leqslant(a_2)_\infty\leqslant\frac{4}{5}$. Similarly curves of a_2 and, where applicable, a_4 against J may be drawn for Tables 3–6. In all such curves a_2 and a_4 tend to some limiting value numerically less than unity for large J. For some of the entries in Table 5 and Table 6 (*a*), $(a_2)_\infty=0$, but in Tables 2, 3 and 5(*a*), i.e. where l_1 and l_2 are both less than 3, $(a_2)_\infty$ is never numerically less than 5/49, so that in all these cases $I(\theta)$ exhibits appreciable anisotropy however large J may be. $(a_4)_\infty$ does not vanish in any of the cases considered, although its numerical value may be as low as 1/49. Inspection of the tables shows that

although most of the factors in the formulae for a_2 and a_4 do not vanish for allowed values of J, there are, however, a few which do, namely $(J-1)$, $(2J-3)$ and $(J-5)$, with the result that certain a_4 vanish at $J=1$ or $\frac{3}{2}$ and certain a_2 vanish at $J=5$. In these cases, which can be quickly spotted in the tables, we have a fortuitous simplification in the angular distribution. A few of the formulae for a_4 have the factors $(J-1)$ and $(2J-3)$ in the denominator, but in such cases the values of J which would give $a_4=\pm\infty$ are not allowed by the angular momentum relations.

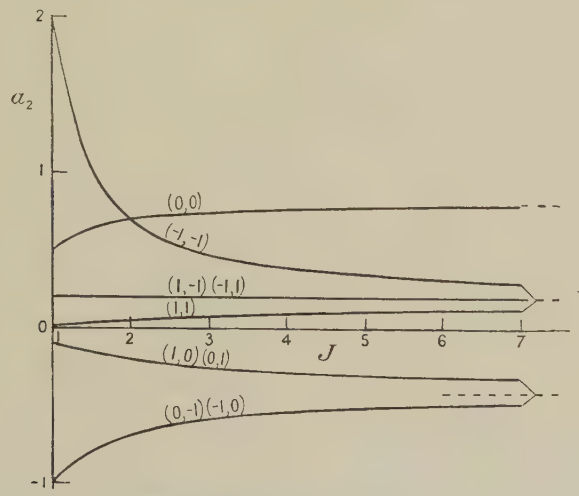

Figure 1. Coefficients a_2 in $1+a_2P_2(\cos\theta)$ as a function of J : $l_1=1=l_2$; curves labelled (Δ, δ) ; asymptotes shown dotted.

Typical distributions of the form $I(\theta)\sim 1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$ are shown in Figures 2 and 3. The first of these is symmetric about a maximum at $\theta=\pi/4$, whilst the second has no turning point in the interval $0<\theta<\pi/2$; the condition for a turning point in this interval is discussed in (iv) below.

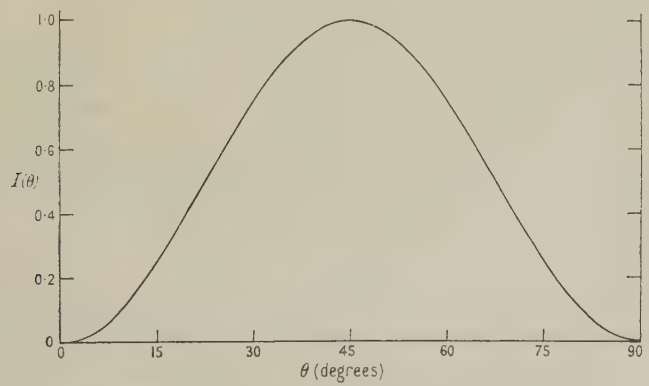

Figure 2. $l_1=2=l_2$, $J+\Delta=0$, $J+\delta=0$,
$I(\theta)\sim 1+\frac{5}{7}P_2(\cos\theta)-\frac{12}{7}P_4(\cos\theta)\sim\cos^2\theta-\cos^4\theta$.

It is interesting to compare the present results with the corresponding ones for γ–γ correlation (Hamilton 1940). Although Hamilton has tabulated coefficients of $\cos^2\theta$ and $\cos^4\theta$, rather than $P_2(\cos\theta)$ and $P_4(\cos\theta)$, it can often be seen by comparing entries in his tables with the corresponding entries in the tables of the

present paper that the γ–γ $I(\theta)$ and the conversion–conversion $I(\theta)$ for a particular double transition have the same general behaviour. This is borne out in comparing the curves in Figure 1 of the present paper with Hamilton's curves for the coefficient of $\cos^2\theta$ in a dipole–dipole type transition. However, in every case the conversion–conversion $I(\theta)$ exhibits stronger anisotropy than the corresponding γ–γ $I(\theta)$; none of the entries in Hamilton's tables permits of 100% anisotropy* in $I(\theta)$, whereas this is certainly possible in the conversion–conversion case, as

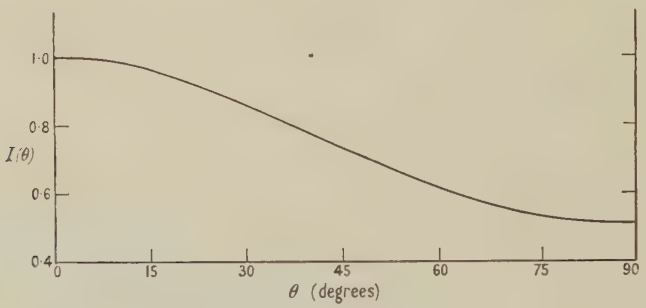

Figure 3. $l_1=3$, $l_2=2$, $J+\Delta=J-3$, $J\geqslant 3$, $J+\delta=J+2$.

$$I(\theta)\sim 1+\frac{10}{21}P_2(\cos\theta)+\frac{3}{77}P_4(\cos\theta).$$

illustrated by Figure 2 and discussed further in (iv) below. It was reasonable to expect the γ–γ directional correlation to be weaker than the conversion–conversion because in the case of γ-quanta it is the polarization which is directly coupled to the nucleus, and the propagation direction enters only indirectly through the fact that it must be perpendicular to the polarization direction. A preliminary report by Falkoff (1948) on calculations of polarization correlation of successive γ-quanta suggests that this will prove stronger on the whole than their directional correlation.

(iv) *Conditions for Zero in* $I(\theta)$ [100% *anisotropy*]

It was thought worth while to work out the conditions that the two types of distribution we have been considering, viz. $I_1(\theta)\sim 1+a_2P_2(\cos\theta)$ and $I_2(\theta)\sim 1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)$, shall have zeroes, since a zero in $I(\theta)$ means 100% anisotropy, and experimentally a large anisotropy should be easier to detect than a small one. We need consider only the interval $0\leqslant\theta\leqslant\pi/2$, since we know $I(\theta)$ is symmetrical about $\pi/2$. Since $I(\theta)$ cannot be negative, zeroes, if they occur, must be at the minima.

$I_1(\theta)$ has either a maximum at $\theta=0$ and a minimum at $\theta=\pi/2$, or vice versa. If the minimum occurs at $\pi/2$, the condition $I_1(\pi/2)=0$ requires $a_2=2$; if it occurs at 0, $I_1(0)=0$ requires $a_2=-1$.

$I_2(\theta)$ also has turning points at $\theta=0$ and $\pi/2$, and will have an additional turning point at some intermediate value of θ given by $\sin^2\theta=6a_2/35a_4+4/7$ when the roots of this equation are real and non-vanishing, i.e. when $-10/3<a_2/a_4<5/2$. In the case where there is no intermediate turning point either $I_2(0)$ or $I_2(\pi/2)$ will be a minimum. $I_2(0)=0$ requires $a_2+a_4=-1$ which, in conjunction with the

* We define percentage anisotropy as $100(I_{\text{greatest}}-I_{\text{least}})/I_{\text{greatest}}$.

condition that a_2/a_4 shall not be within the range $-10/3$ to $5/2$, sets the following individual limits on a_2 and a_4:

$$a_2/a_4 \leqslant -10/3: \quad -10/7 \leqslant a_2 < -1, \qquad 0 < a_4 \leqslant 3/7$$
$$a_2/a_4 \geqslant 5/2: \quad -1 < a_2 \leqslant -5/7, \quad -2/7 \leqslant a_4 < 0.$$

$I_2(\pi/2)=0$ requires $4a_2-3a_4=8$ which, in conjunction with $a_2/a_4 \leqslant -10/3$ or $\geqslant 5/2$, sets the following limits:

$$a_2/a_4 \leqslant -10/3: \quad 80/49 \leqslant a_2 < 2, \quad -24/49 \leqslant a_4 < 0$$
$$a_2/a_4 \geqslant 5/2: \quad 2 < a_2 \leqslant 20/7, \quad 0 < a_4 \leqslant 8/7.$$

If a maximum or minimum does occur between $\theta=0$ and $\pi/2$, $I_2(0)$ and $I_2(\pi/2)$ will be either both minima or both maxima as the case may be; in either event $I_2(0) \gtrless I_2(\pi/2)$ according as $12a_2 \gtrless -5a_4$. Let us take first the equality sign and examine the conditions for the vanishing of $I_2(\theta)$. If there are minima at 0 and $\pi/2$, the vanishing point must occur here and $I_2(0)=I_2(\pi/2)=0$ give the conditions $a_2=5/7$, $a_4=-12/7$; if on the other hand there are maxima at $\theta=0$ and $\pi/2$, with a minimum at some point $\theta=\alpha$ in between, the condition that $I_2(\alpha)=0$ is $10a_4(7+a_2-3a_4)=9a_2^2$, which together with $12a_2=-5a_4$ gives $a_2=-40/49$, $a_4=96/49$.

Next let us take $I_2(0)>I_2(\pi/2)$. In this case vanishing may occur either at $\theta=\alpha$ or at $\theta=\pi/2$, but clearly not at $\theta=0$. Individual limits consistent with the vanishing of $I_2(\theta)$ may be set to a_2 and a_4 by using in turn the conditions for $I_2(\alpha)=0$, $I_2(\pi/2)=0$ together with $12a_2>-5a_4$ and the general condition $-10/3<a_2/a_4<5/2$ for a turning point at $\theta=\alpha$. These limits are as follows:

$$I_2(\alpha)=0[0<\alpha<\pi/2]: \quad -40/49<a_2<7/(6\sqrt{(3/10)}-1)\simeq 3{\cdot}06, \quad 8/7<a_4<18/7$$
$$I_2(\pi/2)=0: \quad 5/7<a_2<80/49 \text{ or } 20/7<a_2<+\infty,$$
$$-12/7<a_4<-24/49 \text{ or } 8/7<a_4<+\infty.$$

Finally we take the case $I_2(0)<I_2(\pi/2)$. Zeroes are now possible at either $\theta=0$ or $\theta=\alpha$ but not at $\theta=\pi/2$. Using in turn the conditions for $I_2(\alpha)=0$, $I_2(0)=0$ together with $12a_2<-5a_4$ and $-10/3<a_2/a_4<5/2$, individual limits may be set to a_2 and a_4 as follows:

$$I_2(\alpha)=0[0<\alpha<\pi/2]: \quad -7/(1+6\sqrt{(3/10)})[\simeq -1{\cdot}63]<a_2<-40/49, 3/7<a_4<96/49$$
$$I_2(0)=0: \quad -\infty<a_2<-10/7 \text{ or } -5/7<a_2<5/7,$$
$$-12/7<a_4<-2/7 \text{ or } 3/7<a_4<+\infty.$$

§ 5. LIMITATIONS OF PRESENT TREATMENT

In this section we discuss to what extent the validity of the foregoing treatment is limited by the initial assumptions listed at the beginning of § 2.

It may first be remarked that assumptions (*a*), (*b*) and (*c*) have the effect of limiting the whole discussion to internal conversion of electric rather than magnetic multipole radiation, for since we exclude spin reversal, the parity change accompanying the ejection of an electron of angular momentum l is just $(-1)^l$—the parity of $Y_{l,m}(\theta,\phi)$, the angular part of the electron wave function—and this is the parity change for an electric 2^l-pole, whereas that for a magnetic 2^l-pole

is $(-1)^{l+1}$. In general any nuclear transition is accompanied by the emission of both electric and magnetic multipole radiations, so that it is clearly necessary to consider their relative importance for internal conversion. This has been done in a recent paper by Drell (1949). It is known that a transition in which the angular momentum changes by ΔJ gives either electric $2^{\Delta J}$-pole with magnetic $2^{\Delta J+1}$-pole radiation, or magnetic $2^{\Delta J}$ with electric $2^{\Delta J+1}$, according to the parity relations*. Drell shows that in the former ('parity allowed') case the number, N_m, of magnetic internal-conversion electrons ejected is quite negligible compared with the number N_e of electric internal conversion electrons (e.g. $(N_m/N_e)_{\text{K-shell}} < 10^{-8}$ for electron energies <1 MEV.), but that in the latter ('parity forbidden') case N_m/N_e may be appreciable. For soft γ-rays in the neighbourhood of the K-electron threshold magnetic conversion approaches a large finite value, whereas electric conversion vanishes at the threshold; consequently in this energy region $(N_m/N_e)_{\text{K-shell}}$ may be large—100 or more—the effect increasing with atomic number Z†. Since the L-shell threshold is one-fourth the K-shell binding energy for the same value of Z, electric L-shell conversion is predominant in this energy region; however, the magnetic conversion in the K-shell may be large enough to contribute significantly to the total conversion coefficient, so that $(N_m/N_e)_{\text{total}}$ cannot be neglected. Such cases are therefore outside the treatment of the present paper.‡

The assumption that the internal conversion electrons come from an s-state means that the present treatment can be applied to the L, M etc. shells only if internal conversion in these shells is predominantly by s-electrons. Since, in general, the mean distance of s-electrons from the nucleus is less than that of p-, d-, etc. electrons one would certainly expect internal conversion to be strongest for s-electrons, but that is not to say that conversion by other electrons is necessarily negligible. Hebb and Nelson (1940) have given non-relativistic formulae (valid for Z less than 40 or 50, and energies very much less than mc^2) for the conversion of electric multipole radiation by the two 2s-electrons and the six 2p-electrons of the L-shell. If we denote by N_{2s} and N_{2p} the total numbers of internal conversion electrons which have been ejected from 2s and 2p states respectively, during the emission of N_q γ-quanta, then Hebb and Nelson's formulae give in effect the quantities N_{2s}/N_q and N_{2p}/N_q in terms of the γ-ray and ejected electron energies, the order l of the electric multipole radiation, and Z. To calculate numerical values of N_{2p}/N_{2s} for definite energies and definite l and Z involves one in a somewhat lengthy evaluation of hypergeometric functions. If we admit energies $\gg mc^2$ the formula certainly gives $N_{2p}/N_{2s} \to 0$, whatever l and Z. Such a step is, of course, logically unjustified, since the formula was derived on the assumption of non-relativistic energies; yet the results of an earlier, relativistic, calculation by Hulme (1932) for the case $l=1$, $Z=84$ suggest that, here at least, the ratios $N_{1s}:N_{2s}:N_{2p}$ are not very sensitive to energy: thus he finds for γ-energy $h\nu \to \infty$, $N_{1s}:N_{2s}:N_{2p} = 6{\cdot}7:1:0{\cdot}053$, and for $h\nu \simeq 1{\cdot}4mc^2$ $N_{1s}:N_{2s} \simeq 7{\cdot}0:1$. Before one can confidently neglect conversion by p-electrons, N_{2p}/N_{2s} should be worked out, using e.g. Hebb and Nelson's formulae, for some $h\nu \ll mc^2$.

* Barring special nuclear symmetries, one is justified in retaining only the lowest allowed multipoles of each type (§ 4 (ii)).

† Drell remarks that this point had been missed in earlier literature, giving rise to some inconsistency in values of magnetic internal-conversion coefficients calculated by different authors.

‡ We must also exclude the rare cases with electric multipole radiation absolutely forbidden by the selection rules, e.g. a $0 \to 1$ transition without parity change.

Finally we have to consider the limitations introduced by neglecting spin reversal. For magnetic multipole radiation internal conversion by s-electrons must be accompanied by spin reversal but, as we have seen above, magnetic conversion is, with certain exceptions, negligible compared with electric conversion. Spin reversal for electric conversion has been shown by Mott and Taylor (1932) to be a small effect for energies less than mc^2. They obtain by a relativistic calculation values of the internal conversion coefficients for the K-shell both with and without spin reversal, calling them ${}_K\alpha_B$ and ${}_K\alpha_A$ respectively, and give curves of these quantities against energy for the case of electric quadrupole radiation and $Z=84$. The following data are taken from their curves: at $h\nu=mc^2$, ${}_K\alpha_B/{}_K\alpha_A\simeq 0{\cdot}2$; at $h\nu=0{\cdot}5mc^2$, ${}_K\alpha_B/{}_K\alpha_A\simeq 0{\cdot}02$; for energies below $0{\cdot}5mc^2$, ${}_K\alpha_B/{}_K\alpha_A$ decreases rapidly.

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Applications of the Peierls–McManus Classical Finite Electron Theory

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ABSTRACT. In the case of an electron in a high-frequency field of small amplitude it is found that the scattering cross section for radiation depends only on the mechanical mass m, instead of the total mass, as in the point electron theory. For the scattering of fast electrons by " protons " the differential cross section for scattering depends on the mass in a more complicated way. Finite values are obtained for the deflection for all values of the impact parameter and for the total energy radiated in the collision.

§ 1. INTRODUCTION

PEIERLS and McManus (McManus 1948) have proposed equations of motion for an electron in an electromagnetic field which can be applied for accelerations and frequencies of any magnitude. Their method depends on the introduction of a Lorentz-invariant form factor $F(\mathbf{R}^2)$ into the interaction term in the relativistic Lagrangian for the motion. This is equivalent to a space–time distribution of electronic charge and also involves a parameter r_0 of the dimensions of a length. The theory yields the non-linear integro-differential equations of motion:

$$m\dot{U}_i - \tfrac{2}{3}e^2(\ddot{U}_i + \dot{\mathbf{U}}^2 U_i) = eU^j \int F_{ij}(\mathbf{z}')F(\mathbf{R}^2)\,d\mathbf{z}'^{(4)}$$
$$+ 2e^2 U^j \int_{-\infty}^{+\infty} [U_j'(z_i - z_i') - U_i'(z_j - z_j')] \frac{dH(\mathbf{R}^2)}{d\mathbf{R}^2}\,ds', \quad \ldots\ldots (1)$$

where $U_j' \equiv U_j(s')$ and $\mathbf{U}$ is the velocity four-vector in relativistic notation; $\mathbf{R}^2 = (z^i - z'^i)(z_i - z_i')$, $i = 0, 1, 2, 3$. The dots denote differentiation with respect to s, the proper time. This means, for example, that $U_1 = u_1/\sqrt{(1-u^2)}$, where u is the magnitude of the velocity and u_1 is a component.

$$F(\mathbf{R}^2) = (2\pi)^{-2} \int g(\mathbf{K}^2) \exp(-i\mathbf{K}.\mathbf{R})\,d\mathbf{K}^{(4)}$$

is the four-dimensional Fourier transform of $g(\mathbf{K}^2)$. The "influence function" $H(\mathbf{R}^2)$ is the transform of $-16\pi^3 g^2(|\mathbf{K}^2|)/\mathbf{K}^2$ (McManus 1948). In this paper it is sufficient to know that $H(\mathbf{R}^2)$ is an even function in $\mathbf{R}^2$ and

$$\int_{-\infty}^{+\infty} H(\mathbf{R}^2)\,d\mathbf{R}^2 = 1.$$

Equations (1) reduce to the Lorentz–Dirac equations when the acceleration and its higher derivatives are small.

In this paper two problems are considered: the motion of an electron in a weak electromagnetic field of arbitrary frequency, and the scattering of fast electrons by a small point charge, whose mass is much greater than the mass of the electron. It is found that in both these cases the non-linear integro-differential equations of motion reduce to linear types, when higher powers of the field

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intensities and charges are neglected. Solution of the former problem shows that, for frequencies large compared to the characteristic frequency, only the mechanical mass need be taken into account in the subsequent motion. The scattering cross section for radiation also depends only on the mechanical mass, which indicates an increase in cross section for higher frequencies when compared with the point electron theory. If a similar result held in a quantized theory, this would enable the mechanical mass to be determined experimentally. So far it has not been possible to formulate such a theory.

Solution of the second problem shows that the scattering formula depends on the mechanical and the electromagnetic mass in a more complicated way. In addition it is found that there is a maximum value of the angle of scattering θ for a particular value of the impact parameter, that is, a maximum value of the differential cross section for some value of θ. Also θ has a finite value, namely zero, for zero impact parameter. In contrast with the divergent value for the total energy radiated by the electron in the collision, using a point theory, the theory presented here yields a finite value of the impact parameter.

§ 2. REDUCTION OF EQUATIONS OF MOTION TO APPROXIMATE LINEAR FORM AND METHOD OF SOLUTION

In the particular applications considered in this paper, the external field is of small amplitude and the motion of the electron is rectilinear. The following approximations are then valid: $s \simeq t$, $z_0 \simeq t$, $z_1 = \zeta$, $z_2 = z_3 = 0$, $\mathbf{R}^2 \simeq (t-t')^2$, the speed of light being taken as $c = 1$. Equation (1) then assumes the form

$$m\ddot{\zeta} - \tfrac{2}{3}e^2\dddot{\zeta} = X(t) + 2e^2 \int_{-\infty}^{+\infty} [(\zeta - \zeta') - \dot{\zeta}'(t-t')]h\{(t-t')^2\}dt', \quad \ldots\ldots (2)$$

where $$h\{(t-t')^2\} \equiv dH(\mathbf{R}^2)/d\mathbf{R}^2;$$

$$\zeta \equiv \zeta(t); \quad \zeta' \equiv \zeta(t'); \quad \dot{\zeta} \equiv \frac{d\zeta}{dt}; \quad \dot{\zeta}' \equiv \frac{d\zeta'}{dt'}; \quad \ddot{\zeta} \equiv \frac{d^2\zeta}{dt^2}; \quad \dddot{\zeta} \equiv \frac{d^3\zeta}{dt^3}$$

$X(t)$ is the "averaged external field",

$$eU^j \int F_{ij}(\mathbf{z}')F(\mathbf{R}^2)\,d\mathbf{z}'^{(4)}.$$

Equation (2) may be solved by means of the Fourier transformation

$$\zeta(t) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} Z(q) \exp(iqt)\,dq. \quad \ldots\ldots (3)$$

It is then easily shown, from equation (2), that

$$Z(q) = \frac{\dfrac{1}{\sqrt{(2\pi)}} \displaystyle\int_{-\infty}^{+\infty} X(t) \exp(-iqt)\,dt}{-mq^2 + \tfrac{2}{3}e^2q^3i + \phi(-q)}, \quad \ldots\ldots (4)$$

with $$\phi(-q) = -A + \sqrt{(2\pi)}B(q) + \sqrt{(2\pi)}iqC(q), \quad \ldots\ldots (5)$$

where, with $u = t - t'$,

$$A = 2e^2 \int_{-\infty}^{+\infty} h(u^2)\,du;$$

$$B(q) = \frac{2e^2}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} h(u^2) \exp(-iqu)\,du;$$

$$C(q) = \frac{2e^2}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} uh(u^2) \exp(-iqu)\,du.$$

The solution of (2) obtained by the above method is valid for any particular form of the "averaging function" $H(u^2)$.

When q is small, equation (5), by expansion of the right-hand side in powers of q, yields

$$\phi(-q) = -e^2q^2 \int_0^\infty H(u^2)\,du = -m_e q^2, \qquad \ldots\ldots (6)$$

where m_e is the electromagnetic mass.

This result is used in the subsequent calculations. The expression $X(t)$ is easily evaluated in the first of the following problems. In the second, however, it is most easily found by using the four-dimensional Fourier representation. This arises from the fact that the properties of the "smearing function" $F(\mathbf{R}^2)$ are simply derived from its transform $g(\mathbf{K}^2)$ (McManus 1948).

§ 3. MOTION OF AN ELECTRON IN A HIGH-FREQUENCY FIELD

Consider an electron irradiated by a plane wave of small amplitude E_0 and frequency $\omega/2\pi$. The electric component may be taken as $E_0 \exp\{i\omega(x_3 - x_0)\}$ in the x_1/-direction. The averaging process then gives $E_0 \exp\{i\omega(z_3 - z_0)\}$, which reduces to $E_0 \exp(-i\omega t)$.

Since the amplitude is small, magnetic force and pressure of radiation may be neglected, so that the motion is rectilinear. Since $X(t) = eE_0 \exp(-i\omega t)$, then

$$\frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} X(t) \exp(-iqt)\,dt = \sqrt{(2\pi)} eE_0 \delta(\omega + q).$$

Hence, from (3) and (4),

$$\zeta(t) = \frac{eE_0 \exp(-i\omega t)}{-m\omega^2 - \frac{2}{3}e^2\omega^3 i + \phi(\omega)}. \qquad \ldots\ldots (7)$$

(i) *ω small*; using (6), (7) yields

$$\zeta(t) = \frac{eE_0 \exp(-i\omega t)}{(-M\omega^2 - \frac{2}{3}e^2\omega^3 i)},$$

where $M = m + m_e$. This is the usual Lorentz value.

(ii) *ω large*: this case is best illustrated by taking a particular function

$$H(\mathbf{R}^2) = (1/2r_0^2) \exp(-|\mathbf{R}^2|/r_0^2).$$

Since $\mathbf{R}^2 \simeq (t - t')^2 = u^2$, we then have $H(u^2) = (1/2r_0^2) \exp(-u^2/r_0^2)$. McManus (1948) discusses the general character of $H(\mathbf{R}^2)$. This particular choice of $H(u^2)$ enables the integrals occurring in the expression for $\phi(\omega)$ to be evaluated exactly by elementary methods, giving

$$\phi(\omega) = -(\sqrt{\pi}e^2/r_0^3) \exp(-\omega^2 r_0^2/4) - (\sqrt{\pi}e^2\omega^2/r_0) \exp(-\omega^2 r_0^2/4) + \sqrt{\pi}e^2/r_0^3. \qquad \ldots\ldots (6')$$

Hence, when $\omega \gg 1/r_0$, the characteristic frequency of the electron, $\phi(\omega) \sim \sqrt{\pi}e^2/r_0^3$, which is negligible compared with $2e^2\omega^3/3$. Thus, from (7), $\zeta(t)$ involves only the rest-mass m, instead of the total mass M, which occurs in the low-frequency case.

The function $$H(u^2) = r_0^2/\{2(r_0^2 + u^2)^2\},$$

which gives the electron a greater extension, leads to a similar result, namely $3\pi e^2/4r_0^3$. More generally, a discussion of the order of magnitude of the integrals

in $\phi(\omega)$, using the fact that $H(u^2)$ is a function of u^2/r_0^2, shows that $\phi(\omega)\sim e^2/r_0^3$ for any even function $H(u^2)$, satisfying the normalization condition.

For intermediate frequencies detailed evaluation for a specific $H(u^2)$ is necessary, as, for example, equation (6′).

Consider now the energy radiated by the electron. It is shown in the Appendix that the standard formula for the average rate of radiation of energy, namely $\frac{1}{3}e^2\ddot{\zeta}\,\ddot{\zeta}^*$, $\ddot{\zeta}$ being complex in this problem, is applicable. Evaluation then yields $e^2E_0^2\omega^4DD^*/3$, where

$$D=1/\{m\omega^2+\tfrac{2}{3}e^2\omega^3 i-\phi(\omega)\}.$$

Thus, for $\omega\gg 1/r_0$, we have

$$e^2E_0^2/\{3(m^2+4e^4\omega^2/9)\}.$$

This means that the scattering cross section for radiation should for high frequencies be greater than the value for a point electron. Quantum effects, however, arise at high frequencies, so that the development of a quantum electrodynamic theory for a finite electron might lead to a different result.

§ 4. SCATTERING OF ELECTRONS BY PROTONS

(The term "proton" is used here in the sense of a heavy particle with a small charge ϵ). The ordinary classical theory of the scattering of fast electrons by protons leads to a divergent result for very small values of the impact parameter when the particle dimensions are neglected. It will be shown that the finite electron theory leads to finite results.

The problem of the scattering of electrons by protons can be converted, by means of a Lorentz transformation, into one in which the electrons are initially at rest and the protons moving. It is convenient to use such a transformation here. Moreover, since the mass of the proton is so much greater than the mass of the electron, the recoil of the proton may be disregarded. For high proton velocities, the proton field will be concentrated in the transverse direction so that the direction of motion of the electron will be approximately at right angles to the direction of motion of the proton. Since the interaction is weak, the speed acquired by the electron will be small.

$$X(t)=eU^j\int F(\mathbf{R}^2)F_{ij}(\mathbf{z}')\,d\mathbf{z}'^{(4)}$$

is most easily calculated in four-dimensional momentum space. $F_{ij}(\mathbf{z}')$ is the field at the point $\mathbf{z}'$ due to the proton at the point $\mathbf{z}$, in four-vector notation. $\mathbf{R}=\mathbf{x}-\mathbf{z}'$, where $\mathbf{x}$ is the four-vector for the electron.

Now
$$F_{ij}(\mathbf{z}')=\frac{\partial A_j(\mathbf{z}')}{\partial z'^i}-\frac{\partial A_i(\mathbf{z}')}{\partial z'^j}$$

and
$$A_j(\mathbf{z}')=(2\pi)^{-2}\int\exp(-i\mathbf{K}'.\mathbf{z}').A_j(\mathbf{K}')\,d\mathbf{K}'^{(4)}.$$

$$\therefore\ F_{ij}(\mathbf{z}')=-(2\pi)^{-2}i\int\{k_i'A_j(\mathbf{K}')-k_j'A_i(\mathbf{K}')\}\exp(-i\mathbf{K}'.\mathbf{z}')\,d\mathbf{K}'^{(4)}$$

Hence, since
$$F(\mathbf{R}^2)=(2\pi)^{-2}\int\exp(-i\mathbf{K}.\mathbf{R}).g(\mathbf{K}^2)\,d\mathbf{K}^{(4)},$$

$X(t)$ becomes

$$-eiU^j\int\exp(-i\mathbf{K}.\mathbf{x}).\{k_iA_j(\mathbf{K})-k_jA_i(\mathbf{K})\}g(\mathbf{K}^2)\,d\mathbf{K}^{(4)}.$$

Now
$$\Box A_i(\mathbf{z}')\equiv\frac{\partial^2A_i}{\partial z_j'\partial z'^j}=4\pi\bar{s}_i(\mathbf{z}'),\qquad\ldots\ldots(8)$$

where $\bar{s}_i(\mathbf{z}') = \epsilon \int_{-\infty}^{+\infty} U_i^P(\sigma) F(\boldsymbol{\rho}^2)\, d\sigma$, with $\boldsymbol{\rho} = \mathbf{z}' - \mathbf{z}(\sigma)$, is the effective charge-current density, and σ is the proper time of the proton, $\mathbf{U}^P(\sigma)$ being its four-velocity.

$$\therefore\ \bar{s}_i(\mathbf{K}) = \epsilon \int_{-\infty}^{+\infty} \exp(i\mathbf{K}.\mathbf{z}).\, U_i^P(\sigma) g(\mathbf{K}^2)\, d\sigma.$$

Now from (8)

$$A_i(\mathbf{K}) = \frac{4\pi \bar{s}_i(\mathbf{K})}{\mathbf{k}^2 - k_0^2} + \tfrac{1}{2}\epsilon i \frac{1}{k}\{\delta(k_0 - |\mathbf{k}|) - \delta(k_0 + |\mathbf{k}|)\}.$$

The second term gives part of the radiation field of the proton. Since the velocity of the proton is constant, this term must vanish. Hence

$$X(t) = 4\pi e\epsilon i U^j \int \exp(-i\mathbf{K}.\mathbf{x}) \exp(i\mathbf{K}.\mathbf{z}).\, \frac{g^2(\mathbf{K}^2)}{\mathbf{K}^2}\{k_i U_j^P - k_j U_i^P\}\, d\sigma\, d\mathbf{K}^{(4)}.$$

Choosing the initial position of the electron as origin, we have $\mathbf{z} = (\tau, -b, v\tau, 0)$; b is the impact parameter; $\mathbf{U}^P = \left(\frac{d\tau}{d\sigma}, 0, v\frac{d\tau}{d\sigma}, 0\right)$; $\mathbf{x} = (t, \zeta, 0, 0)$; $\mathbf{K} = (k_0, k_1, k_2, k_3)$; $\mathbf{K}^2 = k_0^2 - \mathbf{k}^2$. The force $X(t)$ then becomes, since $U^0 \simeq 1$ and the terms with $i = 1, 2, 3$ are negligible,

$$4\pi e\epsilon i \int \exp\{-i(k_0 t - k_1\zeta)\}.\exp(ik_1 b).\exp\{i\tau(k_0 - k_2 v)\}.\, k_1 \frac{g(\mathbf{K}^2)}{\mathbf{K}^2}\, d\tau\, d\mathbf{K}^{(4)}.$$

Integration with respect to τ and k_0 yields, writing $\exp(ik_1\zeta) \simeq 1$,

$$8\pi^2 e\epsilon i \int \frac{g^2[-\{k_1^2 + (1-v^2)k_2^2 + k_3^2\}]}{[-\{k_1^2 + (1-v^2)k_2^2 + k_3^2\}]}\, k_1 \exp(ik_1 b).\exp(-ik_2 vt)\, dk_1\, dk_2\, dk_3.$$

Thus a continuous frequency spectrum enters into this problem.

$$\frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} X(t) \exp(-iqt)\, dt$$

after integration with respect to t and k_2', where $k_2' = k_2 v$, becomes

$$\frac{8\pi^2 e\epsilon \sqrt{(2\pi)}}{v} i \int \frac{g^2[\ ]}{[\ ]}\, k_1 \exp(ik_1 b)\, dk_1\, dk_3,$$

where

$$[\ ] \equiv [-\{k_1^2 + k_3^2 + \frac{1-v^2}{v^2} q^2\}].$$

Hence

$$\zeta(t) = c' \int_{-\infty}^{+\infty} \frac{f(q)\exp(iqt)}{-mq^2 + \frac{2}{3}e^2 q^3 i + \phi(-q)}\, dq, \qquad \ldots\ldots (9)$$

where $c' = \dfrac{8\pi^2 e\epsilon i}{v}$ and $f(q) = \int \dfrac{g^2[\ ]}{[\ ]}\, k_1 \exp(ik_1 b).\, dk_1\, dk_3$.

Now the velocity acquired by the electron may be written as

$$\dot{\zeta} = \int_{-\infty}^{+\infty} \ddot{\zeta}\, dt = \int\int_{-\infty}^{+\infty} \frac{c' q^2 f(q)\exp(iqt)}{mq^2 - \phi(-q) - \frac{2}{3}e^2 q^3 i}\, dq\, dt$$

$$= 2\pi c' \int_{-\infty}^{+\infty} \frac{q^2 f(q)\delta(q)}{mq^2 - \phi(-q) - \frac{2}{3}e^2 q^3 i}\, dq.$$

Now $\phi(-q) \to -m_e q^2$ for q small.

$$\therefore\ \dot{\zeta} = \frac{2\pi c'}{M} \int \frac{g^2\{-(k_1^2+k_3^2)\}}{\{-(k_1^2+k_3^2)\}} k_1 \exp(ik_1 b)\, dk_1\, dk_3, \quad M = m + m_e.$$

$$\therefore\ \dot{\zeta} = -\frac{2\pi c'}{M} \int_0^\infty g^2(-\kappa^2)\, d\kappa \int_0^{2\pi} \exp(ib\kappa \cos\theta) \cdot \cos\theta\, d\theta,$$

where (κ, θ) are plane polar coordinates. Making use of

$$J_0(b\kappa) = \frac{1}{2\pi} \int_0^{2\pi} \exp(ib\kappa \cos\theta)\, d\theta,$$

it is found that

$$\dot{\zeta} = -\frac{32\pi^4 e\epsilon}{Mb} \int_0^\infty g^2(-\kappa^2) \frac{\partial}{\partial \kappa} \{J_0(b\kappa)\}\, d\kappa. \qquad \ldots\ldots (10)$$

Equation (10) could be evaluated for a particular $g(-\kappa^2)$. It is of more interest to obtain the magnitude of $\dot{\zeta}$ for a general function $g(-\kappa^2)$. Two extreme cases are those for very large and very small values of the impact parameter, that is, $b \gg r_0$ and $b \ll r_0$, respectively.

(I) $b \gg r_0$:

g is an even function in κ^2 and is in fact a function of $\kappa^2 r_0^2$. It is effectively non-zero if $\kappa < 1/r_0$. Now $J_0(b\kappa)$ has the asymptotic form

$$\sqrt{\left(\frac{2}{\pi}\right)} \frac{\cos(b\kappa - \pi/4)}{\sqrt{(b\kappa)}}.$$

Thus for $b \gg r_0$ the main contribution will come from $b\kappa$ small, i.e. $\kappa \sim 0$. We may thus replace $g^2(\kappa^2)$ by $g^2(0)$.

$$\therefore\ \dot{\zeta} = -\frac{32\pi^4 e\epsilon}{bMv} g^2(0)[J_0(b\kappa)]_0^\infty;$$

$$\therefore\ \dot{\zeta} = \frac{2e\epsilon}{bMv}, \quad \text{since } g(0) = (2\pi)^{-2} \quad \text{(McManus 1948)}.$$

Therefore, in the rest system of the proton,

$$\dot{\zeta} = \frac{2e\epsilon}{bMv(1-v^2)^{\frac{1}{2}}}.$$

This is the ordinary classical value.

(II) $b \ll r_0$:

Here $b\kappa \ll 1$. Hence

$$J_0(b\kappa) \simeq 1 - \tfrac{1}{4} b^2 \kappa^2 \qquad \ldots\ldots (11)$$

and

$$\dot{\zeta} = \frac{32\pi^4 e\epsilon b}{2Mv} \int_0^\infty \kappa g^2(\kappa^2)\, d\kappa = \frac{be\epsilon(2\pi)^4}{2Mv} \int_0^\infty g^2(\kappa^2 r_0^2)\, d\kappa^2$$

$$= \frac{be\epsilon}{2Mv} \frac{1}{r_0^2} \int_0^\infty \mathcal{G}^2(u^2)\, du^2, \quad \text{where } \mathcal{G}(0) = 1.$$

$$= \text{constant} \times \frac{be\epsilon}{2Mvr_0^2}.$$

Thus, in the rest system of the proton, we have

$$\dot{\zeta} \propto \frac{be\epsilon}{2Mr_0^2 v(1-v^2)^{\frac{1}{2}}},$$

whence $\dot{\zeta} \to 0$ when $b \to 0$.

For intermediate values of b it is necessary to choose a particular form of $g(\kappa^2)$ in order to carry out the integration. It is possible, however, to deduce from the above results the general form of the differential scattering cross section. The graph of θ, the angle of scattering, in terms of b has the form indicated in Figure 1.

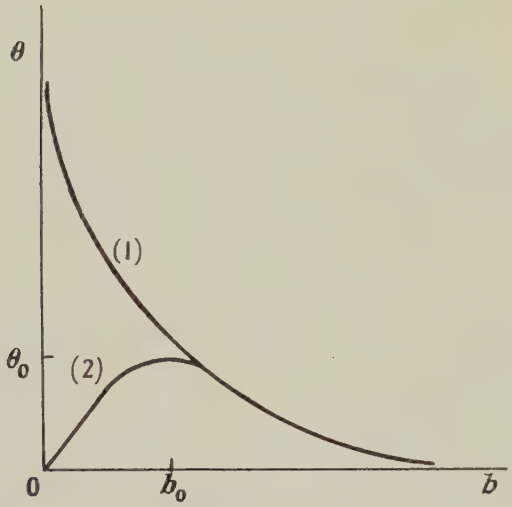

Figure 1. Qualitative graphs of deflection θ against impact parameter b, (1) in point electron theory, (2) in finite electron theory.

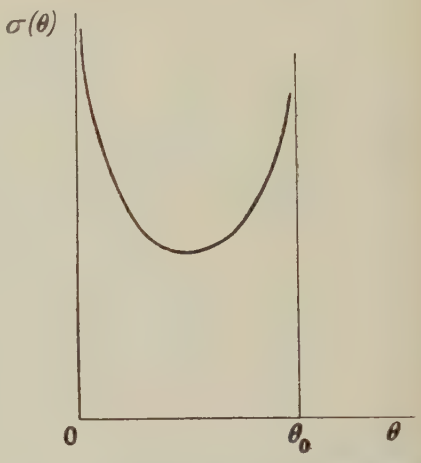

Figure 2. Qualitative graph of differential cross section $\sigma(\theta)$ against angle of scattering θ, in finite electron theory.

Thus θ will have a maximum value θ_0 for some value b_0 of the impact parameter.

Now
$$\sigma(\theta) \propto \frac{b(\theta)}{\theta}\frac{db}{d\theta}$$

and, near θ_0,
$$\theta \simeq \theta_0 + \tfrac{1}{2}\frac{d^2\theta}{db^2}(b-b_0)^2.$$

Hence
$$\left|\frac{db}{d\theta}\right| = 1 \Big/ \sqrt{\left|\left\{2(\theta-\theta_0)\frac{d^2\theta}{db^2}\right\}\right|}.$$

$$\therefore \quad \sigma(\theta) \to \infty, \text{ as } \theta \to \theta_0.$$

A qualitative graph of $\sigma(\theta)$ is shown in Figure 2.

§ 5. RADIATION EMITTED BY SCATTERED ELECTRON

The theorem proved in the Appendix indicates that the classical radiation formula, namely, $\frac{2}{3}e^2\ddot{\zeta}\,\ddot{\zeta}^*$ or $\frac{2}{3}e^2|\ddot{\zeta}|^2$, since ζ in this problem is real, also holds in this case. In this problem it is not very easy to evaluate the radiation spectrum, but the total energy radiated by the electron in the collision may be calculated simply. The radiated energy is

$$\int_{-\infty}^{+\infty} \tfrac{2}{3}e^2\ddot{\zeta}\,\ddot{\zeta}^*\,dt. \qquad \ldots\ldots (12)$$

From (9), $\zeta(t)$ may be written in the form $\int_{-\infty}^{+\infty} \psi(q) \exp(iqt)\, dq$, where

$$\psi(q) = \frac{c' f(q)}{-mq^2 + \frac{2}{3}e^2 q^3 i + \phi(-q)}.$$

Equation (12) then becomes

$$\frac{2e^2}{3} \int_{-\infty}^{+\infty} dq\, dq' \psi(q) \psi^*(q') q^2 q'^2 \int_{-\infty}^{+\infty} \exp\{i(q-q')t\}\, dt.$$

i.e.

$$\frac{4\pi e^2}{3} \int_{-\infty}^{+\infty} q^4 |\psi(q)|^2\, dq,$$

which is equal to

$$c'^2 \frac{4\pi e^2}{3} \int_{-\infty}^{+\infty} \frac{q^4}{|mq^2 - \phi(-q) - \frac{2}{3}e^2 q^3 i|^2} |f(q)|^2\, dq. \qquad \ldots\ldots (13)$$

$|f(q)|$ may be transformed into the integral (cf. equation (10))

$$\frac{2\pi}{b} \left| \int_0^{\infty} \frac{g^2(\kappa^2 + p^2)}{\kappa^2 + p^2} \kappa^2 \frac{\partial}{\partial \kappa} \{J_0(b\kappa)\}\, d\kappa \right|, \qquad \ldots\ldots (14)$$

where $p^2 = q^2(1-v^2)/v^2$.

Case I:

Now for $b \gg r_0$ we should expect to obtain the value of the energy radiated by a point electron, that is, $g^2(\kappa^2 + p^2)$ should be replaced by $g^2(0) = (2\pi)^{-4}$. Evaluation of (14) gives in this case

$$2\pi p K_1(bp),$$

since $$\int_0^{\infty} \frac{\kappa^2}{\kappa^2 + p^2} \frac{\partial}{\partial \kappa} \{J_0(b\kappa)\}\, d\kappa = -2p^2 \int_0^{\infty} J_0(b\kappa) \frac{\kappa}{(\kappa^2 + p^2)^2} d\kappa = bpK_1(bp),$$

where $K_1(bp)$ is a modified Bessel function in Watson's (1944) notation. In the following calculation we make use of the Fourier transform of $PK_1(P)$, namely

$$\frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} \sqrt{\left(\frac{\pi}{2}\right)} \frac{\exp(iPx)}{(x^2+1)^{3/2}} dx,$$

which reduces

$$\int_0^{\infty} P^2 K_1^2(P)\, dP \quad \text{to} \quad \frac{\pi}{4} \int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^3} = \frac{3\pi^2}{32}$$

by elementary integration. Equation (13) then becomes

$$\frac{8}{3\pi} \frac{e^4 \epsilon^2}{v^2} \int_0^{\infty} \frac{q^4 p^2}{[\{mq^2 - \phi(-q)\}^2 + \frac{4}{9}e^4 q^6]} \{K_1(bp)\}^2\, dq. \qquad \ldots\ldots (15)$$

Consideration of the form of the Bessel function K_1 shows that the main contribution of this integral arises when q is small. Hence $\phi(-q) \to -m_e q^2$. Equation (15) then gives

$$\frac{8}{3\pi} \frac{e^4 \epsilon^2}{v(1-v^2)^{\frac{1}{2}} b^3 M^2} \int_0^{\infty} P^2 K^2(P)\, dP, \quad M = m + m_e,$$

with $P = bp = bq(1-v^2)^{\frac{1}{2}}/v$.

The energy radiated is thus equal to

$$\frac{\pi}{4} \frac{e^4 \epsilon^2}{M^2} \frac{1}{b^3 v(1-v^2)^{\frac{1}{2}}}.$$

In the rest-system of the proton this becomes

$$\frac{\pi}{4}\frac{e^4\epsilon^2}{b^3vM^2}\frac{1}{1-v^2},$$

or, in ordinary units, $\pi r^3Mc^3\alpha^2/4b^3v$, where $r=e^2/Mc^2$, $\alpha=(1-v^2)^{-\frac{1}{2}}$, and ϵ is taken equal to e when b is large. This is the usual result given by the point electron theory (Jánossy 1948). The value given by this formula diverges as $b\to 0$.

Case II: $b \ll r_0$.

For $b \ll r_0$ the function $g(\kappa^2+p^2)$ plays a decisive part.

Using the approximation (11) for $J_0(b\kappa)$, we may write

$$|f(q)| = \frac{\pi b}{2}\left|\int_{p^2}^{\infty} g^2(P^2r_0^2)\{1-p^2/P^2\}\,dP^2\right|,$$

where $P^2=\kappa^2+p^2$, $p=q(1-v^2)^{\frac{1}{2}}/v$; therefore the energy radiated is

$$\frac{e^4\epsilon^2b^2}{v^2}(2\pi)^7\int_0^{\infty}\frac{q^4\,dq}{[\{mq^2-\phi(-q)\}^2+\frac{4}{9}e^4q^6]}\left|\int_{p^2}^{\infty} g^2(P^2r_0^2)\{1-p^2/P^2\}\,dP^2\right|^2. \qquad \ldots\ldots (16)$$

The function $g(P^2r_0^2)$ vanishes unless $P<1/r_0$. Also when q is large, i.e. p, P large, the term $(1-p^2/P^2)$ also becomes very small. The main contribution of the integral (16) will thus arise when q is small.

Neglecting the p^2/P^2 term will give a value greater than the exact value. We then have

$$\frac{e^4\epsilon^2b^2}{3v^2}\cdot\frac{1}{2\pi M^2}\int_0^{\infty} dq\left|\int_{p^2}^{\infty}\mathcal{G}^2(P^2r_0^2)\,dP^2\right|^2, \qquad \ldots\ldots (17)$$

where $\mathcal{G}(0)=1$.

To obtain the magnitude of this expression, consider $\mathcal{G}(P^2r_0^2)=\exp(-P^2r_0^2)$. Evaluation of (17) leads to the result $e^4\epsilon^2b^2/\{M^2r_0^5v(1-v^2)^{\frac{1}{2}}96\sqrt{\pi}\}$. Thus in the rest-system of the proton the energy radiated is $\{e^4\epsilon^2b^2/M^2r_0^5v(1-v^2)\}\times$ numerical constant. Thus when $b\to 0$, the energy radiated tends to zero. However, in the above calculation the displacement of the electron in the direction of the incident proton has been neglected, and hence, also, the energy radiated in this motion. This radiation is a second-order effect, but in the limiting case of $b=0$ would give a finite non-zero contribution. In this case the collision is head-on and the value is easily calculated. Thus, in contrast with the point electron theory, the above theory gives a finite value for the radiated energy for any value of the impact parameter b.

§6. CLOSE COLLISIONS FOR RELATIVISTIC ENERGIES

In the case of close collisions, that is, $b\sim r_0$, the momentum acquired by the electron in the direction of the incident proton may be treated qualitatively in the following way. The Coulomb interaction is at most $e\epsilon/r_0^2$, and this acts approximately for a time r_0/v. Hence the momentum acquired by the electron is of the order $e\epsilon/r_0v$. If this is negligible in comparison with $mv(1-v^2)^{-\frac{1}{2}}$, where m is the electron mass, then $e\epsilon/r_0$ must be less than $mv^2(1-v^2)^{-\frac{1}{2}}$. This means that the total energy must be much greater than the self-energy. This is only true for relativistic speeds, that is, v of the order of c, with $c=1$. It is of course possible

to evaluate the velocity in the forward direction exactly by the method already used. This will not, however, yield much more than that obtained by the qualitative argument.

McManus has shown that the effective field of a particle of charge ϵ is given by

$$\overline{F}_{ij}(\mathbf{x}) = \epsilon \int_{-\infty}^{+\infty} \left[U_j \frac{\partial}{\partial x^i} - U_i \frac{\partial}{\partial x^j} \right] \mathrm{H}\{\mathbf{x} - \mathbf{z}(s)\}\, ds + \text{radiation term.}$$

Thus for a proton moving with constant speed, only the first term arises. From the form of H it is then evident that the effective field is reduced for distances of order of magnitude of r_0 or less and is unaffected for distances greater than r_0. Thus the forward momentum acquired by the electron will be relatively small for high proton speeds.

ACKNOWLEDGMENTS

The author wishes to thank Professor Peierls for suggesting the problem and for many valuable discussions, and also Dr. McManus for the use of his thesis.

APPENDIX

The expression

$$\mathrm{X}(r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \chi(k) \exp(ikr)\, dk \qquad \ldots\ldots (a)$$

is easily evaluated by the theory of residues (Whittaker and Watson 1935), the contributions coming from the poles of $\chi(k)$.

It is shown in the following calculation that the result may be generalized for the case of a four-dimensional integral.

The electromagnetic potentials $A_i(\mathbf{x})$, $i = 0, 1, 2, 3$, at the point $\mathbf{x}$, due to an electron of charge e at the point $\mathbf{z}$, are given by

$$A_i(\mathbf{x}) = (2\pi)^{-2} \int \frac{4\pi \bar{s}_i(\boldsymbol{k}, k_0)}{\boldsymbol{k}^2 - k_0^2} \exp(-i\mathbf{K}.\mathbf{x})\, d\mathbf{K}^{(4)} + \frac{e}{2} \int U_i(s) \Delta(\mathbf{R})\, ds, \qquad \ldots\ldots (b)$$

where

$$\bar{s}_i(\mathbf{k}, k_0) = e \int_{-\infty}^{+\infty} \exp(i\mathbf{K}.\mathbf{z}) U_i(s) g(\mathbf{K}^2)\, ds,$$

and

$$\Delta(\mathbf{R}) = \frac{1}{r} [\delta\{(t-t') - r\} - \delta\{(t-t') + r\}],\ r^2 = (x_1 - z_1)^2 + (x_2 - z_2)^2 + (x_3 - z_3)^2.$$

Consider the first integral of (b), denoting it by $A^{(1)}(\mathbf{x})$. In particular, consider the case where

$$\zeta = D \exp(-i\omega t'), \quad \mathbf{z} = (t, \zeta, 0, 0).$$

Then

$$A_1^{(1)}(\mathbf{x}) = (e/\pi) \int \frac{g(k_0^2 - \boldsymbol{k}^2)}{\boldsymbol{k}^2 - k_0^2} \exp(i\mathbf{k}.\mathbf{r}) \exp(-ik_0 t + ik_0 t') \frac{d\zeta}{dt'} d\mathbf{k}^{(3)} dk_0\, dt'.$$

Integration with respect to t' and k_0 gives

$$A_1^{(1)}(\mathbf{x}) = \exp(-i\omega t) 2i\omega e D \int \frac{g(\omega^2 - \boldsymbol{k}^2)}{\omega^2 - \boldsymbol{k}^2} \exp(i\mathbf{k}.\mathbf{r}).\, d\mathbf{k}^{(3)}.$$

Using polar coordinates and integrating over the angles yields

$$\frac{4\pi De}{r} \exp(-i\omega t) \int_{-\infty}^{+\infty} \frac{k}{\omega^2 - k^2} g(\omega^2 - k^2) \exp(ikr)\, dk. \qquad \ldots\ldots (c)$$

This is an integral of the same type as equation (*a*). Poles are at points $k = \pm\omega$. Thus the contributions come from $\mathbf{k}^2 = k_0^2$, i.e. $g(\mathbf{K}^2) = g(0) = (2\pi)^{-2}$. It is fairly simple to deduce from this that $A_1^{(1)}(\mathbf{x})$ may be written in the form

$$\frac{e}{2}\int_{-\infty}^{+\infty} U_1(s)\frac{1}{r}\left[\delta\{(t-t')+r\}+\delta\{(t-t')-r\}\right]ds.$$

Hence

$$A_1(\mathbf{x}) = \int_{-\infty}^{+\infty} U_1(s)\frac{1}{r}\delta\{(t-t')-r\}\,ds,$$

from (*b*). It can be shown in the same way that $A_0(\mathbf{x})$ may be written in a similar form. Then the ordinary form of the retarded potential is valid for a charged particle of finite size, when $r \gg r_0$.

The self-field may then be written as

$$F_{ij}^{(s)}(\mathbf{x}) = e\int_{-\infty}^{+\infty}\left(U_j\frac{\partial}{\partial x^i} - U_i\frac{\partial}{\partial x^j}\right)\frac{1}{r}\delta\{(t-t')-r\}\,ds,$$

thus yielding the standard radiation formula.

In § 4 we had

$$\zeta(t) = \int_{-\infty}^{+\infty}\psi(q)\exp(iqt)\,dq.$$

This gives a continuous spectrum of poles on the real axis. Thus, in this case too, the contributions come from the poles given by $\mathbf{k}^2 = \mathbf{k}_0^2$, so that the radiation formula is also valid in this case.

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Perturbation Theory in Neutron Multiplication Problems

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ABSTRACT. The perturbation theory for neutron diffusion problems in the one group theory is developed systematically by expansion in the system of orthogonal solutions of the integral equation in Part I of the paper.

The multiplication of a neutron source inside a fissile system is discussed. The multiplication factor for a near critical system is derived as a function of the position of the source; isotropic sources are considered in Part II and anisotropic sources in Part III.

Part IV is concerned with a few practical applications of the theory.

INTRODUCTION

THIS report presents the perturbation theory for the calculation of critical sizes of fissile assemblies for small changes in the nuclear properties of the assembly. We confine ourselves to one group theory (all neutrons have the same velocity) and to isotropic nuclear processes. Then a fissile assembly is characterized by two quantities, which in general can be functions of the spatial position: the inverse mean free path $\alpha(r)$ and the number $\beta(r)$ of scattered neutrons and fission neutrons emerging from unit path of one neutron.

The problem of a change in β is closely related to the multiplication problem for the neutrons from an isotropic source inside a subcritical fissile assembly. These two problems have been considered by various authors in unpublished papers, in particular by Davison and Fuchs in Birmingham, by Feynman and Serber at Los Alamos, and probably by workers in other fields with which the author did not have personal contact.

The main purpose of the present paper is an extension of the theory to the multiplication problem for anisotropic sources, which permits an application of the perturbation theory to change in the mean free path.

Since the earlier work is not easily accessible and since no systematic account of the general theory exists, the multiplication problem for isotropic sources and an outline of the perturbation theory for exchanges in β is also presented.

I. GENERAL THEORY

1. *The Integral Equation.*

Let $K(r,r')/v$ be the primary neutron density at the point r, due to an isotropic point source of unit strength at the point r'. We shall count all nuclear processes as removal from the primary beam, so that for a homogeneous medium

$$K(r,r')=\exp(-|r-r'|\alpha)/4\pi|r-r'|^2. \qquad \ldots\ldots(1.1)$$

Here α is the inverse mean free path of the neutron. In an inhomogeneous system K is more complicated. However, K will always be symmetric in r and r', since a neutron is just as likely to penetrate from a point r' to the point r as from r to r'. Hence

$$K(r,r')=K(r',r). \qquad \ldots\ldots(1.2)$$

The symmetry property is retained if scattering processes without change of energy are not considered as removal from the primary beam, and the general theory developed below applies to this case as well.

Let there be a source per unit volume of strength $q(r')$ at the point r'. In a subcritical system this source distribution will give rise to a neutron density $n(r)$, which satisfies the equation

$$n(r) = \int \beta(r')K(r,r')n(r')\,dV' + \int \frac{q(r')}{v} K(r,r')\,dV'. \quad \ldots\ldots (1.3)$$

Here v is the velocity of the neutrons. The second term represents all the primary neutron densities due to the neutron sources. The first term accounts for the secondary contributions of the neutrons removed from the primary beam as well as tertiary contributions and so on. The total number of neutrons removed per unit time from any beam in a unit volume at r' is $\alpha(r')n(r')v$. Let now β/α be the number of neutrons emerging from the volume per neutron removed. We then have a source $\beta(r')vn(r')$ and multiplication with K/v yields the contribution of this source to $n(r)$.

2. *Eigensolutions of the Homogeneous Integral Equation*

Let us consider the homogeneous integral equation, obtained by omitting the source terms. This equation has solutions $f_i(r)$ for an infinite number of eigenvalues λ_i. Thus

$$f_i(r) = \lambda_i \int \beta(r')K(r,r')f_i(r')\,dV'. \quad \ldots\ldots (2.1)$$

The $f_i(r)$ form a complete set of functions which are orthogonal to the set $\beta(r)f_i(r)$. They may be normalized so that

$$\int \beta(r)f_i(r)f_j(r)\,dV = \delta_{ij}. \quad \ldots\ldots (2.2)$$

The kernel K can be expanded in the form $K(r,r') = \Sigma_i f_i(r)f_i(r')/\lambda_i$. Any arbitrary function $g(r)$ can be expanded in the form $g(r) = \Sigma_i [g]_i f_i(r)$ with coefficients $[g]_i = \int \beta(r)g(r)f_i(r)\,dV$.

3. *Variation Theory*

Let us vary the system under consideration to a slight extent. Such a variation may be expressed as a change $\Delta\beta$ and ΔK. As a result the λ_i and f_i will change by amounts $\Delta\lambda_i$ and Δf_i. Variation of the equation (2.1) yields

$$\Delta f_i(r) = \lambda_i \int \beta(r')K(r,r')\Delta f_i(r')\,dV' + \Delta\lambda_i \int \beta(r')K(r,r')f_i(r')$$
$$+ \lambda_i \int [\Delta\beta(r')K(r,r') + \beta(r')\Delta K(r,r')]f_i(r')\,dV'. \quad \ldots\ldots (3\ 1)$$

Multiplying with $\beta(r)f_i(r)$ and integrating we find with the help of the integral equation (2.1) and the symmetry property of the kernel K that the integral of the left-hand side of equation (3.1) cancels the integral of the first term on the right-hand side. The remaining terms yield the equation

$$\frac{\Delta\lambda_i}{\lambda_i} + \int \Delta\beta(r)f_i(r)^2\,dV + \lambda_i \int f_i(r)\beta(r)\Delta K(r,r')\beta(r')f_i(r')\,dV\,dV' = 0. \quad \ldots\ldots (3.2)$$

This equation is particularly useful if there is no change in the kernel so that the last term vanishes. The resultant equation has been used by Davison and Fuchs (1943) to calculate the effect of a thin reflector on the critical size. It is also used extensively in pile calculations, where it is known as the Statistical Weight Theorem (see for example Goodman (1947)). It is also closely related to the Feynman theorem, which states:

If two critical systems with the same kernel K differ only in the values of $\beta(r)$, then

$$\int [\beta_1(r)-\beta_2(r)]n_1(r)n_2(r)\,dV=0, \qquad \ldots\ldots(3.3)$$

where n_1 and n_2 are the neutron densities in the two systems satisfying the integral equations

$$\left.\begin{aligned} n_1(r)&=\int \beta_1(r')K(r,r')n_1(r')\,dV',\\ n_2(r)&=\int \beta_2(r')K(r,r')n_2(r')\,dV'. \end{aligned}\right\} \qquad \ldots\ldots(3.4)$$

On multiplying the first equation by $\beta_2 n_2$ and the second by $\beta_1 n_1$ and integrating, the equation (3.3) is immediately evident.

4. *The Eigenvalue λ of a Near Critical System*

For a critical system the lowest eigenvalue λ is unity. For a subcritical system λ is greater than unity and it is convenient to use λ as a measure of the criticality of the system.

We may change a critical system into a subcritical system by removing the fissile material from a volume ΔV and replacing it, for example, by reflector material. We shall confine ourselves to the special case when the mean free path in the fissile material and reflector are identical. (The more general case requires the methods developed later to deal with anisotropic sources.) The method also applies to a bare fissile system, which can be regarded as being surrounded by an ideal absorber ($\beta=0$). For equal mean free paths ΔK vanishes and

$$\frac{\Delta\lambda}{\lambda}=-\int \Delta\beta(r)f(r)^2\,dV. \qquad \ldots\ldots(4.1)$$

Furthermore, β is changed in the volume ΔV by the difference of β in core and reflector, but remains the same everywhere else. Hence

$$\frac{\Delta\lambda}{\lambda}=(\beta_c-\beta_r)\Delta V\overline{f(r)^2}, \qquad \ldots\ldots(4.2)$$

where the bar denotes an average over the volume ΔV and the subscripts c, r refer to the core and reflector.

In the case of a spherical assembly we assume that the system is reduced below critical by slightly reducing the radius a of the fissile material. Observing that $\Delta M/M=\Delta V/V_c$ one finds the following relation between the change in λ and the change ΔM in the mass of fissile material

$$\frac{\Delta\lambda}{\lambda}=(\beta_c-\beta_r)f(a)^2V_c\Delta M/M. \qquad \ldots\ldots(4.3)$$

II. MULTIPLICATION PROBLEMS*

5. *The Neutron Density from a Source in a Subcritical System*

We now return to the inhomogeneous integral equation (1.3). Expand the neutron density n in terms of the proper functions f_i

$$n(r)=\Sigma_i[n]_i f_i(r); \qquad [n]_i=\int \beta(r)n(r)f_i(r)\,dV. \qquad \ldots\ldots(5.1)$$

With the help of the integral equation (1.3), one finds

$$\lambda_i[n]_i=[n]_i+\frac{1}{v}\int q(r)f_i(r)\,dV.$$

The second term can be considered the expansion coefficient of q/β. Thus

$$(\lambda_i-1)[n]_i=[q/\beta]_i/v \qquad \ldots\ldots(5.2)$$

and

$$vn(r)=\Sigma_i\frac{1}{\lambda_i-1}[q/\beta]_i f_i(r), \qquad \ldots\ldots(5.3)$$

$$[q/\beta]_i=\int q(r)f_i(r)\,dV. \qquad \ldots\ldots(5.4)$$

For a nearly critical system the first proper value is nearly unity and therefore only the first term in the expansion (5.3) need be considered. Omitting the suffix, we have

$$vn(r)=\frac{1}{\Delta\lambda}[q/\beta]f(r). \qquad \ldots\ldots(5.5)$$

6. *The Multiplication*

Let us calculate now the number of neutrons which escape from the system. In a stationary state this number is identical with the difference between the number created and the number lost in the system, which is given by

$$E=\int vn(r)[\beta(r)-\alpha(r)]\,dV+\int q(r)\,dV. \qquad \ldots\ldots(6.1)$$

For vn we substitute (5.3); q we write in the form $\beta q/\beta$ and we expand q/β. Then

$$E=\Sigma_i[q/\beta]_i\left\{\frac{1}{\lambda_i-1}\int f_i(r)\{\beta(r)-\alpha(r)\}\,dV+\int\beta(r)f_i(r)\,dV\right\}$$

$$=\Sigma_i\frac{1}{\lambda_i-1}[q/\beta]_i\int\{\lambda_i\beta(r)-\alpha(r)\}f_i(r)\,dV. \qquad \ldots\ldots(6.2)$$

The multiplication m is defined by $m=E/Q$, where Q is the integrated source of neutrons.

The equation for the multiplication is a special case of the neutron current across a closed boundary. Equation (6.2) evidently holds in general for the current $E(S)$ across any closed surface S, provided the integral is extended only over the volume inside the surface S.

$$E(S)=\Sigma_i\frac{1}{\lambda_i-1}[q/\beta]_i\int_{(S)}\{\lambda_i(\beta r)-\alpha(r)\}f_i(r)\,dV. \qquad \ldots\ldots(6.3)$$

* Some of the material presented in Part II is a reconstruction of calculations done by Serber at Los Alamos.

7. *The Multiplication in a Near Critical System*

In a near critical system again only the first term in the expansion need be considered. Then we find

$$E=Qm=\frac{1}{\Delta\lambda}\int q(r)f(r)\,dV\int\{\beta(r)-\alpha(r)\}f(r)\,dV. \qquad \ldots\ldots(7.1)$$

For a spherical assembly we use (4.3) and find $m=pM/\Delta M$, where

$$p=\frac{\int q(r)f(r)\,dV}{f(a)\int q(r)\,dV}\,\frac{\int\{\beta(r)-\alpha(r)\}f(r)\,dV}{(\beta_c-\beta_r)f(a)V_c}. \qquad \ldots\ldots(7.2)$$

It might be noted that in this form the function f need not be normalized.

The first factor in (7.2) is the ratio of the average neutron density weighted by the source distribution to the neutron density at the core surface. We may write it symbolically in the form $\bar{n}_q/n(a)$. The number of neutrons escaping from the system can also be written in the form of an integral over the surface

$$E=\int\overline{\cos}\,vn(r)\,dS, \qquad \ldots\ldots(7.3)$$

where $\overline{\cos}$ is the average direction cosine of the neutrons relative to the normal to the surface. Comparison with the above formula yields

$$\int\overline{\cos}\,n(r)\,dS=\int(\beta-\alpha)n(r)\,dV. \qquad \ldots\ldots(7.4)$$

8. *The Infinite Perfect Reflector*

The second factor in (7.2) is particularly simple if the reflector only scatters but does not absorb neutrons. Then $\beta-\alpha$ vanishes in the reflector. If we assume further that the mean free path and hence α in core and reflector are equal, (7.2) reduces to

$$p=\bar{n}_q\bar{n}_c/n(a)^2, \qquad \ldots\ldots(8.1)$$

where $\bar{n}_c$ denotes the average neutron density in the core.

For simplicity let us apply diffusion theory to the calculation of the neutron density $n(r)$ in the critical system and let us assume an infinite reflector. The net radial current through the surface of a sphere of radius r in the reflector is equal to the number of neutrons created in the core minus those absorbed in the core. Hence

$$-D\,dn/dr=\int_{\text{core}}(\beta-\alpha)vn(r)\,dV/4\pi r^2, \qquad \ldots\ldots(8.2)$$

where the diffusion coefficient $D=v/3\alpha$. Thus

$$n=\frac{3}{4\pi r}\alpha(\beta-\alpha)\int_{\text{core}}n\,dV=a^3\alpha(\beta-\alpha)\bar{n}_c/r. \qquad \ldots\ldots(8.3)$$

Hence $d(nr)/dr=0$. In the core $n=\sin(kr)/kr$, where $k^2=3\alpha(\beta-\alpha)$. Continuity of n and $D\,dn/dr$ implies that the gradient of rn be continuous. Since it vanishes in the reflector, we require $ka=\frac{1}{2}\pi$. Further, $\bar{n}_c=3[\sin(ak)-ak\cos(ak)]/(ak)^3$. Thus

$$n(0):\bar{n}_c:n(a)=\tfrac{1}{2}\pi:(12/\pi^2):1. \qquad \ldots\ldots(8.4)$$

Hence we have:

$$\left.\begin{aligned} &\text{For a point source at the centre: } p = 6/\pi = 1{\cdot}91. \\ &\text{For a constant source in the core: } p = 144/\pi^4 = 1{\cdot}49. \\ &\text{For a source at the surface of the core: } p = 12/\pi^2 = 1{\cdot}22. \end{aligned}\right\} \quad \ldots\ldots (8.5)$$

For a source distribution in the core proportional to the neutron density one finds $p = \overline{n_c}^2/n(a)^2$, which is easily shown to be equal to 3/2.

III. MULTIPLICATION FOR ANISOTROPIC SOURCE

9. *The Angular Velocity Distribution*

Let $N(r, \Omega)$ be the density of neutrons at the point r with velocities having directions within the element of solid angle $d\Omega$. N satisfies the Boltzmann equation or transport equation.

$$(\Omega \,.\, \mathrm{grad}) N(r, \Omega) + \alpha(r) N(r, \Omega) = \frac{\beta(r)}{4\pi} n(r), \qquad \ldots\ldots (9.1)$$

$$n(r) = \int N(r, \Omega)\, d\Omega. \qquad \ldots\ldots (9.2)$$

Assuming $n(r)$ to be known, we can obtain N by integration of (9.1),

$$N(r, \Omega) = \frac{1}{4\pi} \int_0^\infty \exp\left[- \int_0^s \alpha\{r - s'\Omega\}\, ds' \right] \beta(r - s\Omega) n(r - s\Omega) s^2\, ds. \qquad \ldots\ldots (9.3)$$

Here α, β are defined to be zero in free space. This equation can be expressed in the form

$$N(r, \Omega) = \int_0^\infty K(r, r - s\Omega) \beta(r - s\Omega) n(r - s\Omega) s^2\, ds. \qquad \ldots\ldots (9.4)$$

Integration over Ω leads to the integral equation for n, since $s^2\, ds\, d\Omega = dV$.

10. *Anisotropic Source*

Let there be a source of distribution such that $q(r, \Omega)\, d\Omega\, dV$ is the rate of emission of neutrons from the volume element dV with directions within the element of solid angle $d\Omega$. We reduce this problem to an isotropic source distribution by considering the neutrons arising from the first collision. These correspond to an isotropic neutron source

$$\left.\begin{aligned} q_1(r) &= 4\pi\beta(r) \int K(r, r') q(r', \Omega)\, dV', \\ \Omega &= (r - r')/|r - r'|. \end{aligned}\right\} \qquad \ldots\ldots (10.1)$$

In general we should now substitute two source terms into our equations corresponding to q and q_1. However, for a near critical system the direct contributions from the source terms are negligible. In this case we can immediately apply the equations derived before. Our problem then reduces to the determination of the coefficient

$$[q/\beta] = \int q_1(r) f(r)\, dV = 4\pi \iint f(r) \beta(r) K(r, r') q(r', \Omega)\, dV\, dV'. \qquad \ldots\ldots (10.2)$$

We integrate first over dV. Using a coordinate system centred on the point r' we have $dV = s^2\, ds\, d\Omega$ and $r = r' + s\Omega$. With (9.4) one finds

$$[q/\beta] = 4\pi \iint N(r', -\Omega) q(r', \Omega)\, dV'\, d\Omega. \qquad \ldots\ldots (10.3)$$

A particularly important case arises if the angular distribution of the source is determined by the angular distribution of the neutron density, i.e.

$$q(r, \Omega) = q(r)N(r, \Omega)/f(r), \qquad \ldots\ldots(10.4)$$

where $q(r)$ is the total strength of the source, then

$$[q/\beta] = \int q(r)f(r)g(r)\,dV, \qquad \ldots\ldots(10.5)$$

which differs from the isotropic case by the factor $g(r)$ defined by

$$g(r) = \frac{4\pi}{f^2(r)} \int N(r, \Omega)N(r, -\Omega)\,d\Omega. \qquad \ldots\ldots(10.6)$$

The function $g(r)$ can be worked out for any system for which the neutron density is known.

For a system with spherical symmetry it is evident that at the centre $g = 1$, and at the boundary of the system $g = 0$. The latter is true for any system with convex boundaries.

IV. APPLICATIONS

11. *Effect of Changes at the Centre of a Spherical System* (Measurement of absorption cross sections)

Let us consider a near critical spherical system with a small hole at the centre. The hole is assumed to be sufficiently small so that the neutron distribution in the hole can be assumed to be isotropic. In that case scattering by a sample introduced into the hole has no effect on the neutron distribution.

If there is some arbitrary distribution of sources in the system, the neutron density is obtained from (5.5),

$$vn_1(r) = \frac{1}{\Delta\lambda}\left[\frac{q}{\beta}\right] f(r) = S_1 f(r). \qquad \ldots\ldots(11.1)$$

The number of neutrons escaping from the system is given by (6.2),

$$E_1 = \frac{1}{\Delta\lambda}\left[\frac{q}{\beta}\right] \int (\beta - \alpha)f(r)\,dV; \qquad \ldots\ldots(11.2)$$

by observing E_1 we can determine $\Delta\lambda$.

Let us now introduce a sample of material into the hole and let β^*, α^* be characteristic for the material. In that case the neutron density changes and will be

$$vn(r) = Sf(r), \qquad \ldots\ldots(11.3)$$

with a coefficient S which we shall determine presently.

Since the effect of scattering can be neglected, the only effect of the sample will be to absorb or produce neutrons. This effect can be regarded as a source of neutrons of strength

$$Q = vn(0)(\beta^* - \alpha^*)V = Sf(0)(\beta^* - \alpha^*)V, \qquad \ldots\ldots(11.4)$$

where V is the volume of the sample. The factor S is now determined by the primary source distribution and the source Q, i.e.

$$S = \frac{vn(r)}{f(r)} = S_1 + \frac{1}{\Delta\lambda}\left[\frac{Q}{\beta}\right] = S_1 + \frac{1}{\Delta\lambda} Sf(0)^2(\beta^* - \alpha^*)V. \qquad \ldots\ldots(11.5)$$

Solving for S we find

$$S = S_1 \Big/ \left[1 - \frac{1}{\Delta\lambda} f(0)^2 (\beta^* - \alpha^*) V \right]. \qquad \ldots\ldots (11.6)$$

An experimental measurement of the ratio S/S_1 allows therefore a determination of $\beta^* - \alpha^*$. The escape of neutrons is evidently proportional to S. Hence $S/S_1 = E/E_1$ can be observed directly.

The result (11.6) could in this particular case have been obtained more directly by application of (3.3). Since scattering does not matter, the empty hole can be replaced by a hole with $\alpha = \beta = \alpha^*$. On placing the sample into the hole the kernel is unchanged and β changes by $\beta^* - \alpha^*$. Hence $\Delta\lambda$ changes by $f(0)^2(\beta^* - \alpha^*)V$. Now E is inversely proportional to $\Delta\lambda$: hence $E_1/E = [\Delta\lambda - f(0)^2(\beta^* - \alpha^*)V]\,\Delta\lambda$, which is identical with (11.6).

Another use of the equation (11.6) is obtained by observing that for a critical system the neutron density from a source becomes infinite. If we arrange that the system be just critical if the hole is filled with fissile material, it follows that the denominator of (11.6) vanishes for $\beta^* = \beta$, $\alpha^* = \alpha$ and therefore $\Delta\lambda = f(0)^2(\beta - \alpha)V$ is a measure of the effect of the hole on the criticality of the system. Combining this equation with (4.3) we find that a hole increases the critical mass by the amount ΔM given by

$$\frac{\Delta M}{M} = \frac{\beta_c - \alpha_c}{\beta_c - \beta_r} \frac{f(0)^2}{f(a)^2} \frac{V}{V_c}. \qquad \ldots\ldots (11.7)$$

This last formula is of course dependent upon the assumption of equal mean free path in core and reflector which was used in order to derive (4.3).

12. *Effect of a Change of α in a Small Region*

In the general case a change of the properties in a small region which changes α affects the kernel $K(r, r')$ for any two points which lie on a straight line passing through the affected region. In that case the evaluation by means of formula (3.3) is more complicated and we follow instead the procedure by means of artificial sources.

Let again the suffix 1 refer to the starting system, for which the formulae (11.1) and (11.2) apply. Let us then change the values of β and α in a small region of volume V at the point r by the amounts $\Delta\beta$ and $\Delta\alpha$ (not necessarily small). The change in $\Delta\beta$ corresponds to an isotropic source of strength

$$Q_1 = vn(r_0)\Delta\beta V = Sf(r_0)\Delta\beta V. \qquad \ldots\ldots (12.1)$$

An increase in α on the other hand implies that fewer neutrons pass through the region without collision and it corresponds to an anisotropic source of strength

$$Q_2 = -SN(r_0, \Omega)\Delta\alpha V. \qquad \ldots\ldots (12.2)$$

Corresponding to (11.5) we now find

$$S = S_1 + \frac{SV}{\Delta\lambda} \{ f(r_0)^2 \Delta\beta - f(r_0)^2 g(r_0) \Delta\alpha \}. \qquad \ldots\ldots (12.3)$$

Hence

$$S = S_1 \Big/ \left\{ 1 - \frac{V f(r_0)^2}{\Delta\lambda} [\Delta\beta - g(r_0)\Delta\alpha] \right\}. \qquad \ldots\ldots (12.4)$$

Evidently these equations are easily generalized for an extended source and they will, for example, allow us to estimate the error introduced by the neglect of scattering for a finite sample placed at the centre of the assembly. This error may not be negligible even for a small sample, since for most materials β is very close to α and therefore $\beta - g\alpha$ may differ appreciably from $\beta - \alpha$, even if g is fairly close to unity.

ACKNOWLEDGMENTS

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Half-lives of ^{141}Ce and ^{169}Yb

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ABSTRACT. The half-lives of ^{141}Ce and ^{169}Yb, produced through slow neutron capture in cerium and ytterbium, have been redetermined as $(33{\cdot}11 \pm 0{\cdot}23)$ days and $(31{\cdot}83 \pm 0{\cdot}21)$ days respectively. The errors appear to be smaller than in previous determinations.

§ 1. INTRODUCTION

Slow neutron capture in cerium produces, in addition to shorter-lived activities, an activity with a half-life of about 30 days. This has been assigned to ^{141}Ce. Slow neutron capture in ytterbium produces, again in addition to shorter-lived activities, an activity with a half-life of about 33 days. This has been assigned to ^{169}Yb (Seaborg and Perlman 1948). During the course of some radioactivity experiments, the decay in activity of neutron-activated cerium and ytterbium has been followed over a lengthy period. It seems of value to report the somewhat precise values which have thus been found for the half-lives of ^{141}Ce and ^{169}Yb, as the errors appear to be smaller than those in earlier determinations.

§ 2. HALF-LIFE OF ^{141}Ce

A sample of 'Specpure' cerium oxide* was irradiated in the low power pile at the Atomic Energy Research Establishment, Harwell, intermittently over a period of three weeks. The decay in activity of a source prepared from the irradiated material was followed using a Geiger counter with a 7 mg/cm^2 window and a standard source-counter geometry. The first point shown on the decay curve (Figure 1) was obtained 11 days after the removal of the sample from the pile. A source of RaE in equilibrium with RaD (22 years half-life) was used to standardize the counter sensitivity, allowance being made for the small but appreciable decay of the RaD. The decay of the cerium source was accurately exponential after 100 days, when the 13-day ^{143}Pr formed by the decay of 36-hour ^{143}Ce had decayed

* Catalogue number J.M.304, Johnson, Matthey and Co. Ltd., London.

away. The exponential portion of the curve in Figure 1 covers about 9 half-lives and the activity decayed steadily, without significant departure from the exponential, until lost in the background. The straight line drawn through the observed points is based on a 'least squares' calculation for the points after and including 165·5 days to ensure that the ^{143}Pr has a negligible influence. The point at 402·9 days is the last point with any significant weight.

The straight line corresponds to a half-life for ^{141}Ce of (33·11 ± 0·23) days.

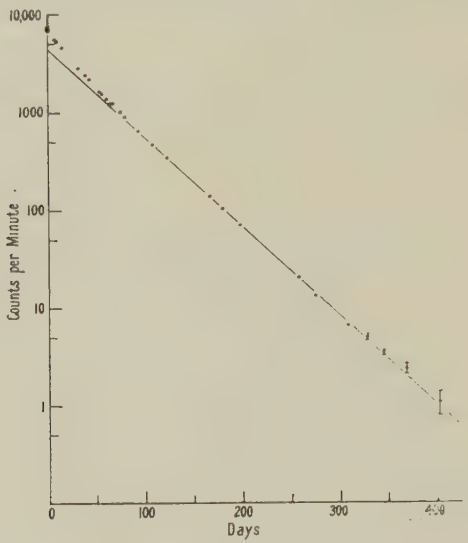

Figure 1. Decay curve of cerium.

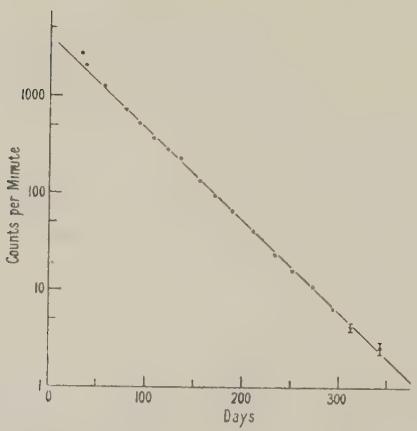

Figure 2. Decay curve of ytterbium.

§ 3. HALF-LIFE OF ^{169}Yb

A sample of 'Specpure' ytterbium oxide* was irradiated in the pile for 4 days. The decay in activity was followed using a procedure similar to that for cerium. Zero time on the decay curve (Figure 2) corresponds to 1 day out of the pile. The decay was accurately exponential after 75 days when the 100-hour ^{175}Yb had

* Catalogue number J.M.319, Johnson, Matthey and Co. Ltd., London.

decayed away. The exponential portion of the curve covers about 8 half-lives and the activity decayed steadily, without significant departure from the exponential, until lost in the background. The straight line drawn through the observed points is based on a 'least squares' calculation for the points after and including 92·8 days to ensure that the ^{175}Yb has a negligible influence. The point at 342·7 days is the last point with any significant weight.

The straight line corresponds to a half-life for ^{169}Yb of (31·83 ± 0·21) days.

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The Decay and Capture of μ-Mesons in Photographic Emulsions

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ABSTRACT. Observations on the decay and capture of μ-mesons in Kodak NT4 and Ilford G5 photographic emulsions are described. It is found, in agreement with other workers, that 63 ± 4% of all μ-mesons stopping in the emulsion decay with the emission of a fast electron, a result which is shown to be in reasonable agreement with theoretical predictions. The emission of *slow* electrons is observed from 7·2 ± 1·3% of all slow mesons, and these are ascribed mainly to an Auger effect accompanying the capture of μ-mesons.

§ 1. INTRODUCTION

It has already been shown that it is possible to detect the decay electron of the μ-meson in the new highly sensitive emulsions (Brown, Camerini, Fowler, Muirhead, Powell and Ritson 1949). In this paper the results of an investigation of the mechanism of stopping and capture or decay of these mesons in the various elements of the emulsion is described. The photographic emulsion is not a very suitable medium for this purpose since it is composed partly of heavy elements, silver and bromine, combined in the form of dielectric crystals, and partly of an amorphous mixture of light elements, carbon, hydrogen, nitrogen and oxygen. Moreover, it has a high stopping power which prevents the recognition of electrons of energy less than 20 kev.

Exposures have been made at the Pic du Midi of batches of plates of the Kodak NT4 and Ilford G5 emulsions, of thicknesses 200, 400 and 600 microns, for 6 days and 12 days. All mesons entering the plate from outside and ending in the emulsion without giving rise to a star or to a secondary meson have been classified as positive or negative μ-mesons (formerly designated as ρ-mesons).

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§ 2. OBSERVATIONS AND FRACTION OF μ-MESONS WHICH DECAY

In Table 1 are given the observed numbers of mesons in the various exposures, separated into mesons which stop giving rise to no observable electron (μ), those which give a fast electron only ($\mu+e$), those giving a fast electron and one or more slow electrons ($\mu+e+A$), and those giving rise only to slow electrons ($\mu+A$).

Table 1

Exposure (days)	Plates	Thickness (microns)	Area (cm^2)	Total No.	μ	$\mu+e$	$\mu+e+A$	$\mu+A$
6	Kodak NT4	400	100	65	18	40	1	6
12	Kodak NT4	400	131	205	78	114	0	13
12	Kodak NT4	200	109	61	24	31	1	5
6	Ilford G5	600	50	82	32	45	0	5

A decay electron will be observed only if it is emitted at a sufficiently small angle to the plane of the emulsion. In order to determine the fraction of decay of electrons which have been missed, the lengths of the tracks of all the decay electrons in plates of the same thickness have been measured. Using the usual geometrical argument, the distribution so obtained has been fitted to the calculated distribution. The correction made in this way shows that failure to observe dipping electrons causes a loss of 10% of the observed number in a 400 micron emulsion. The number of tracks available in the other plates is not sufficient to repeat the measurements for 200 and 600 micron plates, but it is assumed that the loss is approximately inversely proportional to the thickness, i.e. 7% for 600 microns and 20% for 200 microns.

In order to find the fraction of mesons giving rise to decay electrons, all mesons giving a fast electron have been put in one group ($\mu+e$) and all without a fast electron in the other group (μ), regardless of the presence or absence of slow electrons. The results after making the geometrical correction are given in Table 2.

Table 2

Plate	Thickness (microns)	Total No. of mesons	$\mu+e$	μ	% of mesons which decay
Kodak NT4	400	270	171	99	63 ± 5
Kodak NT4	200	61	38	23	62 ± 10
Ilford G5	600	82	48	34	59 ± 9

The percentage of decay in the different groups of plates is consistent within the experimental errors, and the results, combined for all the plates, give a total number of 413 mesons, of which 257 decay (i.e. $62\pm4\%$) and 156 are captured.

Approximately 2·5% of these particles will be negative π-mesons which are captured by the nucleus and give rise to no visible star. This is deduced from the fact that the number of negative π-mesons giving stars is 10% of the number of ρ-mesons, and the results of experiments at Berkeley which show that 25% of the negative π-mesons ending in emulsions insensitive to electrons give no ionizing particles. Thus of the 413 mesons which stop in the emulsion, 409 will be μ-mesons; of these, 257 decay (i.e. $63\pm4\%$) and 152 are captured, a result in good agreement with the Bristol group (Brown *et al.* 1949).

The existence of a positive excess of slow mesons is not definitely established for energies of the order of 10 Mev. which are important in the photographic plate.

Preliminary reports from Franzinetti on the magnetic sandwich method indicate that positive and negative μ-mesons are present in approximately equal numbers. Thus, in the absence of evidence to the contrary, it will be assumed that the numbers are equal. Then of the 409 μ-mesons the 204 ± 14 which are positive must all decay, so that of the negatives 53 ± 7 ($26 \pm 3\%$) decay and 74% are captured. If it is assumed that among the various elements of the gelatine the probability of stopping a meson is proportional to Z, and the probability of capture is proportional to Z^4 (Wheeler 1947), the fraction of negative mesons stopped in gelatine which decay is about 80%. On the same reasoning all the negative mesons stopped in AgBr will be captured. Thus the decay of the negative mesons requires that 30% are stopped in the gelatine. A rough calculation of the relative number stopped, assuming that the stopping power at low energies is still proportional to the number of electrons per unit path, gives a value of 33% for gelatine. This agreement may be considered satisfactory.

§ 3. MESONS WHICH ARE CAPTURED WITH EMISSION OF SLOW ELECTRONS

As indicated in Table 1, the emission of slow electrons has been observed from the ends of $7{\cdot}2 \pm 1{\cdot}3\%$ of the meson tracks, and these are ascribed mainly to an Auger effect accompanying the capture of μ-mesons. A detailed analysis shows that this proportion implies an actual occurrence of more than 20% in the cases in which a negative meson stops in the heavy elements of the emulsion.

Although all the slow electrons are ascribed in this paper to μ-mesons, it is possible that an appreciable fraction could be due to negative π-mesons which are captured without producing visible stars. Only experiments with artificially produced mesons can give the exact value of this fraction, but certainly not more than one-third of the effect we have observed could be ascribed to this cause. The actual proportion is, in fact, probably much smaller than one-third because this value has been obtained on the assumption that *all* the captured negative π-mesons emit slow electrons. If the emission of slow electrons by these negative π-mesons is the same as for star-producing σ-mesons, the contribution would fall from one-third to one-seventh.

The extent of the loss of slow electrons emitted due to the failure to recognize them as such is very difficult to estimate. These electrons are highly scattered and, therefore, it is not possible to determine their point of origin unless the density of grains in the vicinity of the end of the meson track is high and even. When of very low energy (20–30 kev.) they can easily be obscured by the track of the meson. When of higher energy (50–150 kev.), the beginning of the electron track is frequently well defined, but it is difficult to follow it to the end if the general density of slow electrons in the neighbourhood is high. Electrons of less than 20 kev. will never be recognized, since their range is too short.

It is essential to establish a rigorous criterion for the admission of a slow electron as originating from the end of a meson, since the probability of a chance juxtaposition of the end of the meson track with the beginning of the track of one of the numerous electrons of the background is appreciable. The criterion adopted is that slow electrons should be admitted as Auger electrons only if the beginning of the track can be certainly identified within 1 micron of the end of the meson. The probability of a chance effect has then to be determined. This is done in two ways. First, the number of slow electron tracks on 6,360 microns of

meson track and 6,700 microns of proton tracks are counted. A total of 90 slow electrons is found, giving a frequency of 1·5% for the chance effect. It is realized that some of these slow electrons may be genuine delta rays ejected by the fast particle, and an attempt has been made to repeat the measurement using the image of a fine wire superposed on that of the emulsion. This gives a frequency of slow electrons of the order of 0·2%. A more direct approach has therefore been adopted. Among the protons ejected from the stars, search has been made for those giving an apparent Auger effect, which in this case must certainly have been spurious. In the exposure of 6 days one such effect has been found in 270 protons and in the 12-day exposure one in 220 protons. The probability of a chance effect can therefore be estimated as not more than 0·5% even in a 12-day exposure.

The results given in Table 1 show a frequency of the Auger effect of 7% of the total number of mesons, i.e. 14% of all negative μ-mesons (neglecting the contribution from negative π-mesons). This figure is certainly a lower limit since the rigorous criterion excludes possibly as many Auger effects as have been accepted. Of these 7%, less than 1% are associated with a fast electron, which strongly

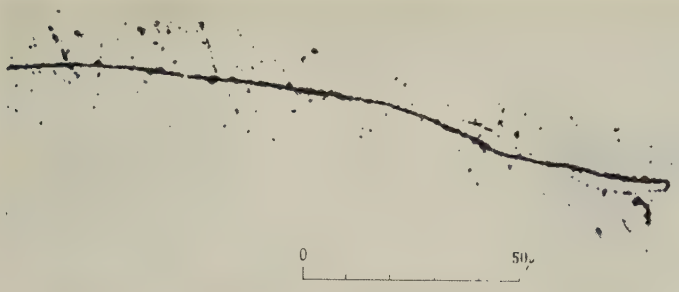
Table 3. Energy of Observable Auger Electrons in the Chief Components of the Emulsion

Element	Energy of Auger Electrons (kev.) Transitions							
	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9
Carbon	78	14·5						
Nitrogen	106	19·7						
Oxygen	139	25·8						
Bromine	2650	494	173	80	43·5	26·1	17·0	
Silver	4780	890	311	144	78·4	47·1	30·7	21·4

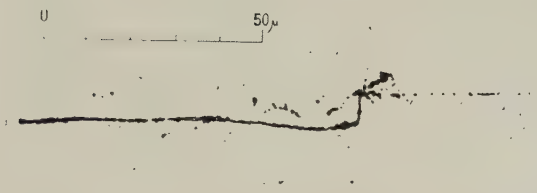
indicates that practically all the Auger electrons are produced in AgBr. From the above estimate of 30% of mesons stopping in gelatine, it follows that of negative mesons stopped in AgBr at least 20% give rise to an observable Auger electron.

The energy of the electrons that should be expected from the Auger effect can be deduced from the position of the orbits of a meson of mass 214 round the nuclei of the various elements of the emulsion. These are given in Table 3. It will be seen that in the gelatine only the last transitions of the meson will give rise to Auger effects observable in the photographic plate (i.e. of energy greater than 20 kev.). According to the theory of Fermi and Teller (1947) it is in the last transitions that the emission of radiation rather than Auger effects become predominant. In the case of AgBr there are several possible transitions before the meson arrives in the region of loss by radiation (for these elements the loss by radiation becomes important for meson orbits whose radii are of the order of Zr_K, r_K being the radius of the meson K orbit corresponding to the atomic number Z). The exact values of the relative probabilities of Auger effect and of radiation on this theory have not, to our knowledge, been calculated.

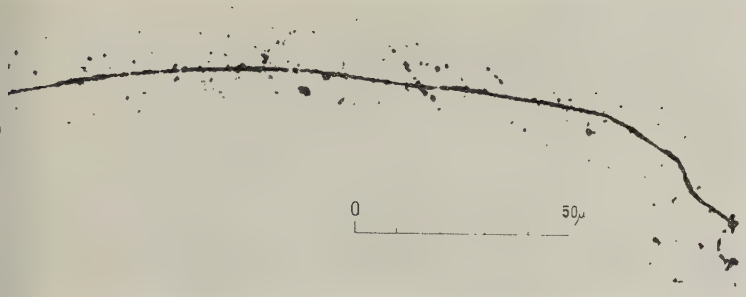
The observed distribution in range of the Auger electrons is given in Figure 1 (*a*). The range–energy relation of slow energy electrons in these emulsions is not yet known with precision and may possibly depend somewhat on the batch, but some idea of the corresponding energies can be obtained from the curve given for Kodak NT2a emulsions. The distribution in energy of the Auger



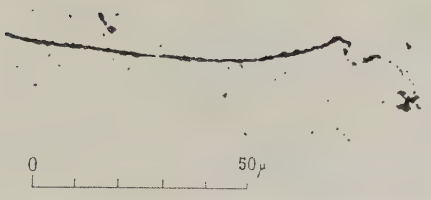
Single slow electron.
Plate: Kodak NT4, 400 μ.
Exposure: 6 days.
Obs.: Eric Shramm.



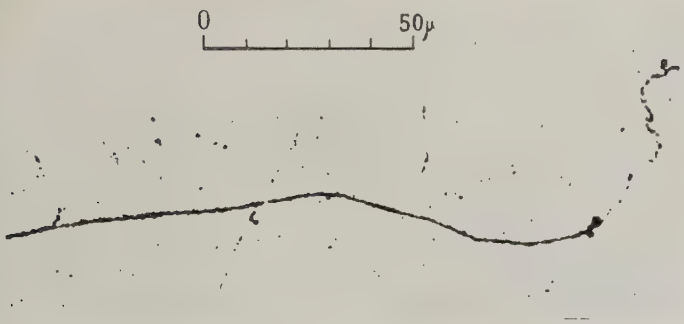
Several slow electrons
and single fast electron.
Plate: Kodak NT4, 400 μ.
Exposure: 6 days.
Observer: Mme Hubert.



Two slow electrons.
Plate: Kodak NT4, 400 μ.
Exposure: 6 days.
Observer: Mme Hubert.



Single slow electron.
Plate: Ilford G5, 600 μ.
Exposure: 6 days.
Observer: Mme Hubert.



Three slow electrons.
Plate: Kodak NT4, 400 μ.
Exposure: 12 days.
Observer: Mme Hubert.

Slow electrons observed at the end of meson tracks.

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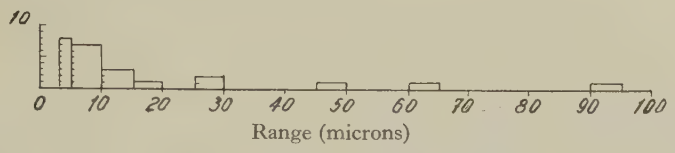
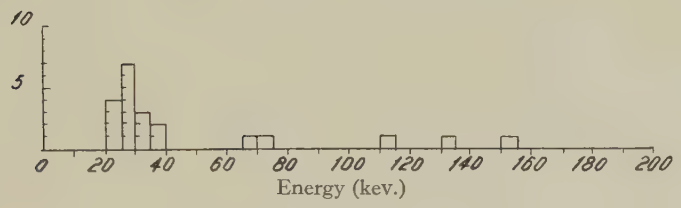
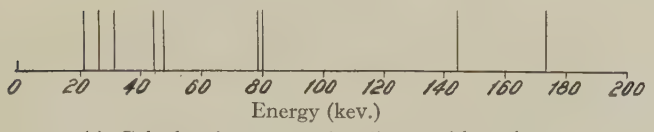
electrons is given in Figure 1 (*b*) and the spectrum calculated for silver and bromine in Figure 1 (*c*). Taking into account the difficulties in recognition and measurement of these tracks and the uncertainty of the range–energy curve, there is reasonable agreement. The absence of well-defined lines in the experimental spectrum can be easily explained by the overlapping of the lines of the various elements; moreover, these lines have probably been broadened by the errors of observation and straggling.



(*a*) Observed number of Auger electrons.



(*b*) Observed number of Auger electrons.



(*c*) Calculated spectrum for silver and bromine.

Figure 1.

The conclusions which can be drawn from these measurements seem to be that the existence of the Auger effect—which should more properly be called internal-conversion effect since the transfers of energy correspond to meson orbits well within the electron cloud—has been demonstrated, and its frequency is not large. This indicates that it is largely suppressed by a more probable competing process such as radiation.

ACKNOWLEDGMENTS

This work, and other work which is in progress, would not have been possible without the cooperation of many people. We therefore wish to thank Dr. Berriman of Kodak Ltd. and Mr. Waller of Ilford Ltd. for kindly supplying us with the special thick plates used. We thank Dr. Rösch, Director of the Observatory of the Pic du Midi, and his staff, who arranged the exposures for us. We also take this opportunity to thank Dr. Taylor of Messrs. Cooke, Troughton and Simms, who furnished us with a special long-distance oil immersion objective, with which tracks at the bottom of the 600-micron plates can be viewed through the backing glass.

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A Contribution to the Theory of Soft X-ray Emission Bands of Sodium

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ABSTRACT. A wave-mechanical treatment of the $L_{II,\ III}$ emission spectrum of sodium is given, taking into account (i) the broadening of the energy levels in the conduction band due to electron collisions (§§ 2–4); (ii) the lowering of the potential energy of the electron about to make transition into an inner shell of an ion, due to electrostatic attraction (§§ 5–6). The first problem is treated in the free electron approximation, using a model suggested by Skinner, and the theoretical curve has a ' tail ', as found experimentally. The calculation shows that a modification of the wave functions of electrons in a free electron model may be required, and this possibility is discussed. In the second problem, the depression in the electron potential is approximated by using a spherical potential well in the neighbourhood of the ion. The first effect proves to be the more important one, and there is good agreement with experiment.

§ 1. INTRODUCTION

In the soft x-ray spectroscopy of solids the experimental results are expressed by plotting the intensity I of the emitted frequency ν against either the frequency or the energy E to which it corresponds (Skinner 1938). The zero of the energy is usually taken at the bottom of the valency electron energy band. If $P(E)$ is the probability of a transition from a level of energy E into the ionized inner shell, and $N(E)$ the density of states of energy E in the band, then $I(E)$ is given by $I(E) \propto \nu^3 P(E) N(E)$. The factor ν^3 is unimportant, as it varies only little over the width of a band.

In the theoretical discussion of these curves (Jones, Mott and Skinner 1934, Matyáš 1948), two factors have not so far been considered, although they are bound to affect the theoretically predicted result. First, in calculating the function $N(E)$, one must take into account the broadening of the individual energy levels by electron collisions in the conduction band. For the treatment of this problem we shall adopt the model suggested by Skinner (1940). The problem here is to calculate the life-time of an electron vacancy in the valency electron energy levels. This life-time is finite, and the level therefore broadened, because, due to electron collisions, an electron from higher up in the band can drop into the vacant level while another valency electron is transferred to a state above the surface of the Fermi distribution. The details of the present calculations show that the initial and final states of the electrons must be such as to ensure energy and momentum conservation. To find the total broadening one must consider all collisions of this type which will lead to the removal of the initial vacancy. This calculation is made in §§ 2–4, and the (I, E) curve for sodium so obtained is drawn in Figure 4. The result reproduces the characteristic ' tail ' of Skinner's experimental curve.

Secondly, in calculating the transition probability $P(E)$, one must take into account the fact that the electron which makes the transition into the inner shell moves in the field of the ionized atom just before the transition is made (cf. also Platt 1946). Thus, in the neighbourhood of the ion the potential of the electron

is lowered, instead of remaining constant as previously assumed. To estimate the effect of this modification, a calculation of $P(E)$ has been made in §§ 5 and 6, using a potential well of specified radius and depth in the neighbourhood of the ion. Theoretical $I(E)$ curves predicted with the modified function $P(E)$ are given in Figure 5. Whereas the first effect causes the $I(E)$ curve to rise more slowly than $\sqrt{E}$, the second effect is seen to cause a rise of the curve at a rate which always exceeds that of $\sqrt{E}$. Both results have been combined in Figure 6, and the agreement with Skinner's experimental curve for the sodium $L_{II,III}$ radiation is satisfactory. The 'bump' near the emission edge remains unaccounted for by these calculations, but it has not been observed by Cady and Tamboulian (1941) who, however, appear to have used a lower resolving power.

In confining attention to a monovalent metal such as sodium in this paper, we do in fact treat the theoretically simplest case, since the function $N(E)$ becomes complicated as soon as the surface of the Fermi distribution approaches the boundary of the first Brillouin zone. Although the theoretical expressions derived in this paper can be applied to all metals for which the $N(E)$ curve is similarly simple, only the $L_{II,III}$ emission of sodium has been treated here.

§ 2. PERTURBATION THEORY FOR THE ELECTRON COLLISIONS

In order to account for the tails obtained in the $L_{II,III}$ emission spectra Skinner (1938, 1940) and Cady and Tamboulian (1941) have introduced the hypothesis of an Auger effect in the conduction band of a metal which, in this particular case, is equivalent to a collision between a pair of electrons. Cady and Tamboulian supposed that part of the energy difference involved in the transition of a valency electron into a vacant x-ray level is taken up by the transition of a second valency electron into a vacant level above the Fermi surface. This mechanism could very greatly increase the width of an emission band, and seems to be less likely than Skinner's mechanism (cf. § 1). As a detailed discussion of the inter-relation between the electron collisions and the emission of x rays is probably very difficult, we shall restrict ourselves here to calculations based on Skinner's model. In this an electron falls from a full conduction band into a vacant x-ray level emitting radiation, so that there is a single vacant level in the final state of the conduction band. This radiation is not monochromatic since, owing to the Auger effect, which is induced by the Coulomb interactions between the electrons, radiationless transitions occur between this state and a state in which there are *two* vacant levels and one electron above the Fermi surface. The assumed final state of the conduction band after emission of the radiation is, therefore, not a stable state but has a finite and short life-time. Hence this state has no precise energy, and the half-width ΔE of the level is related to the life-time Δt of the assumed final state of the conduction band by the relation $\Delta E \sim \hbar/\Delta t$. Each energy level in the band can then be replaced by an appropriately broadened level. Finally, by integrating the probability contributions from all broadened levels to a level of fixed energy E, we obtain a new density of states function which reproduces approximately the tail found by experiment. Throughout the calculation we need consider only the Fermi gas formed by the 3s electrons of sodium.

The perturbation which determines the transition probability is the difference between the complete Hamiltonian, which includes the Coulomb interaction between all electrons, and the Hamiltonian of the Hartree–Fock approximation in which this interaction is replaced by the effect of a self-consistent field. The

perturbation consists therefore of the Coulomb interaction plus a sum of terms each of which depends on the coordinates of only one electron. For the wave functions of the initial and final states of the metal we shall use determinants of normalized plane waves, which implies that we are treating the problem in the one-electron approximation and are assuming the valency electrons of sodium to be free. If the total number of valency electrons is $2n$, each determinant is equivalent to a product of two determinants, each having n rows and columns. One determinant involves the n functions which refer to one type of spin, whereas the other involves the functions which refer to the other type of spin (cf. Wigner and Seitz 1934). Attention will be confined in the first place to collisions of electrons having opposite spins, and it can then be seen that the matrix element U_{if} (connecting the initial and the final state) of the perturbation will not involve an exchange term. Suffixes i and f denote the initial and final states respectively.

Let V be the volume of the metal, and let the two electrons partaking in the collision be denoted by suffixes 1 and 2, with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ in the initial state and $\mathbf{k}_1'$ and $\mathbf{k}_2'$ in the final state. It may then be verified that only the term e^2/r_{12} of the perturbation, and no other, contributes to U_{if}, which is given by

$$U_{\mathrm{if}} = e^2 V^{-2} \int r_{12}^{-1} \exp\{i[(\mathbf{k}_1' - \mathbf{k}_1).\mathbf{r}_1 + (\mathbf{k}_2' - \mathbf{k}_2).\mathbf{r}_2]\} d\tau_1 \, d\tau_2$$

$$= \begin{cases} 4\pi e^2/Vg^2 & \text{if } \mathbf{k}_1 - \mathbf{k}_1' = \mathbf{k}_2' - \mathbf{k}_2, \\ 0 & \text{otherwise,} \end{cases} \qquad \ldots\ldots(1)$$

where $\mathbf{g} \equiv \mathbf{k}_1 - \mathbf{k}_1' = \mathbf{k}_2' - \mathbf{k}_2$. Thus the matrix element vanishes unless the momentum is conserved, and we note particularly that *all* Coulomb interactions between the electrons have been taken into account in (1).

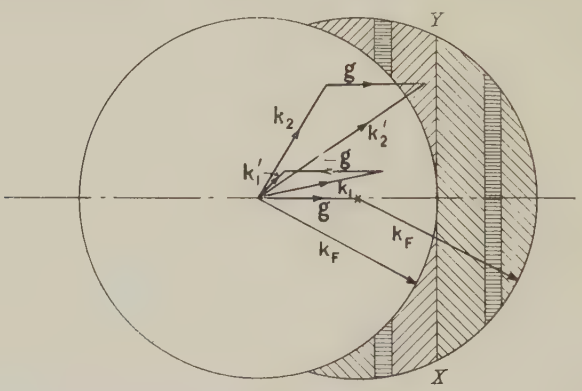

Figure 1. Integration over all wave vectors of electron 2. The figure shows the permissible volume (shaded) for the independently variable vector $\mathbf{k}_2'$ if $\mathbf{k}_1$ and $\mathbf{k}_1'$ (and therefore $\mathbf{g}$) are fixed. The nature of the volume element, which is also shown, changes at the plane XY so that the integration decomposes into two parts.

If at time $t = 0$ the metal is in a state with total energy E_{i}, the probability of a transition to a state of total energy E_{f} after a time t is given by (Heitler 1944, p. 88)

$$P_{\mathrm{if}} = (2t^2 U_{\mathrm{if}}^2/\hbar^2)[(1 - \cos x)/x^2], \qquad \ldots\ldots(2)$$

where $x = t(E_{\mathrm{f}} - E_{\mathrm{i}})/\hbar$. It is required to calculate the life-time of the unoccupied level specified by the wave vector $\mathbf{k}_1'$. We have therefore to consider transitions from all occupied levels $\mathbf{k}_1$ with energy greater than the energy of the level $\mathbf{k}_1'$,

and all possible transitions of the second electron from an occupied level $\mathbf{k}_2$ to a level $\mathbf{k}_2'$ above the Fermi surface. P_{if} must therefore be summed over all possible vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. This implies a double volume integration which is carried out in the next section, where it will be shown that the total transition probability into the vacant level at $\mathbf{k}_1'$ diverges as $1/g$ for small g.

§ 3. THE INTEGRATION OF EQUATION (2) OVER ALL PERMISSIBLE COLLISIONS

To sum the probabilities, note that for a fixed change in the momentum of either electron in the collision, i.e. for fixed $\mathbf{g}$, x can take a range of values, and we shall now determine the number of possible collisions when $\mathbf{g}$ is fixed and x lies between x and $x+dx$. The number of states per unit volume of k-space is $V/8\pi^3$, where V is the volume of the metal. Since we are calculating the life-time of the vacancy specified by $\mathbf{k}_1'$, this vector remains constant in the whole calculation. Hence, for given $\mathbf{g}$, it follows that $\mathbf{k}_1$ is also constant, and from the definition of x we have

$$x=(t\hbar/m)(\mathbf{k}_2'-\mathbf{k}_1).\mathbf{g}. \qquad \ldots\ldots(3)$$

Thus x varies by virtue of the variation of $\mathbf{k}_2'$ only. We have now to find the permissible volume of $\mathbf{k}_2'$-space. From (3) it follows that for given x and $\mathbf{g}$, $\mathbf{k}_2'$ must lie on a plane at right angles to $\mathbf{g}$. The area of this plane is limited by the condition that for the unoccupied state $\mathbf{k}_2'$, $k_2'>k_F$, and that for the occupied state $\mathbf{k}_2'$, $k_2<k_F$. If $\mathbf{k}_2'.\mathbf{g}\leqslant k_F g$ we obtain an annulus of area $\pi(2\mathbf{k}_2'.\mathbf{g}-g^2)$, whilst if $\mathbf{k}_2'.\mathbf{g}>k_F g$ we obtain a complete disc. The range for which $\mathbf{k}_2'.\mathbf{g}>k_F g$ corresponds to collisions in which energy is not conserved, and can therefore be neglected. The limits x_1 and x_2 of the integration over the significant range are determined by the greatest and least value of $\mathbf{k}_2'.\mathbf{g}$, which are, respectively, $k_F g$ and $g^2/2$. Hence

$$x_1=(t\hbar/m)(\tfrac{1}{2}g^2-\mathbf{k}_1.\mathbf{g}); \qquad x_2=(t\hbar/m)(k_F g-\mathbf{k}_1.\mathbf{g}).$$

Since $k_1>k_1'$, x_1 is always negative; x_2 is always positive if $\mathbf{k}_1$ lies below the Fermi surface. For the thickness of the volume element we find from (3)

$$g^{-1}d(\mathbf{k}_2'.\mathbf{g})=(m/t\hbar g)\,dx.$$

The number of collisions for which g is a non-zero constant, and x lies between x and $x+dx$, is now given by

$$(V/8\pi^3)\pi(2\mathbf{k}_2'.\mathbf{g}-g^2)(m/t\hbar g)\,dx.$$

The sum over all P_{if} subject to the above conditions becomes

$$(4me^4/V\hbar^3)\,t/g^5\int_{x_1}^{x_2}[(2mx/t\hbar)+2\mathbf{k}_1.\mathbf{g}-g^2](1-\cos x)x^{-2}\,dx. \qquad \ldots\ldots(4)$$

Evaluating (4) in the usual way, we obtain for the probability that in time t one electron has made a transition from an initial state $\mathbf{k}_1$ to a final state $\mathbf{k}_1'$, all possible transitions of the second electron being taken into account,

$$(4\pi me^4/V\hbar^3)g^{-5}(2\mathbf{k}_1'.\mathbf{g}+g^2)t. \qquad \ldots\ldots(5)$$

Hence the probability of any electron making a transition into the vacant state $\mathbf{k}_1'$ in time t, due to a collision between two electrons, is obtained by summing (5) over all possible values of $\mathbf{g}$. The lower limit of the integration over g should

be $g=0$ in any theory based on the Bloch model. However, the integral of (5) with respect to g diverges as $1/g$ at this limit. A difficulty of a similar nature has recently been found by Heisenberg, and will be discussed in § 3.

To see the nature of the divergence, we transfer the origin to the fixed point $\mathbf{k}_1'$ ($k_1'>0$) and take as volume element $2\pi \sin\theta\, d\theta\, g^2 dg$, where θ is the angle between $\mathbf{k}_1'$ and $\mathbf{g}$. Hence the main contribution to the transition probability, which comes from the region of small g, is given by

$$(2me^4/\pi\hbar^3)k_1' t \int \cos\theta \sin\theta\, d\theta \int g^{-2} dg, \qquad \ldots\ldots(6)$$

where the volume element has been multiplied by $V/8\pi^3$ to give the number of states contained in it.

It is important to notice that only those electrons can make transitions whose initial energy exceeds the energy of the vacant state $\mathbf{k}_1'$, since the accompanying transition of the second electron is one which absorbs energy. Thus for a given value of g the smallest permissible value of k_1 is k_1', and in this case the angle between $\mathbf{k}_1'$ and $\mathbf{g}$ has its greatest value, which is

$$\theta_1 = \tfrac{1}{2}\pi + \sin^{-1}(g/2k_1'). \qquad \ldots\ldots(7)$$

The integral over θ is therefore $\frac{1}{2}\left|\sin^2\theta\right|_0^{\theta_1}$ which, by (7), is approximately $\frac{1}{2}$ for small g. If, in order to obtain a finite life-time for the vacant state, we follow Heisenberg's suggestion and assume that those electrons lying in a spherical shell of k-space bounded by the energy surfaces E_F and $E_F-\epsilon$ ($\epsilon \ll E_F$) do not participate in the collisions, then our previous calculations are not materially affected and the smallest allowable value of g is given by $2g/k_F=\epsilon/E_F$. Thus we obtain from (6) for the probability per unit time of an electron making a transition into the state $\mathbf{k}_1'$ due to collisions between two electrons

$$P=(4/\pi\hbar)(k_1'/k_F)(E_F/\epsilon)W,$$

where W is the ionization energy of a normal hydrogen atom. Writing now k for k_1', and denoting by $\Delta(k)$ the half-width of an energy level on that spherical shell in k-space for which the wave vector has the magnitude k, we obtain for $k>0$ (Heitler 1944, p. 113)

$$\Delta(k) \sim \hbar P = 17{\cdot}2\,(k/k_F)(E_F/\epsilon) \text{ ev.} \qquad \ldots\ldots(8)$$

For sodium $E_F \sim 3$ ev., and if we choose $\epsilon \sim 1/40$ ev. we see that the theoretical value of the line width can reach several hundred electron volts; even if we choose $E_F/\epsilon \sim 10$, expression (8) is in contradiction to the experimental tails found by Skinner, which are only of the order of 1 ev. long.

§ 4. DISCUSSION OF THE PRECEDING CALCULATION OF THE LINE WIDTH

The introduction of ϵ was suggested by some considerations of Heisenberg's (1947) in connection with his recent theory of superconductivity. He calculated the second-order perturbation energy which involved matrix elements which correspond to transitions from a state represented by a full Fermi sphere to excited states represented by a sphere with two electrons outside it, the transition being induced by that term of the Hamiltonian which represents the mutual interaction of the electrons. This energy was found to diverge as $\log p$, where $\pm\mathbf{p}$ is the change in momentum of the two electrons. From this comparatively slow

divergence Heisenberg concluded that, whereas for a free electron model the ordinary free electron wave function may still be at least qualitatively correct for electrons inside the Fermi sphere, this is no longer so for the electrons which are in a thin layer at the surface of the sphere. If in the present calculations the electrons in this layer are assumed not to take part in the collisions, one would then expect, on Heisenberg's view, a reasonable result for the line width. That this expectation is not fulfilled is due to the fact that expression (8) for the line width diverges as $1/\epsilon$, i.e. much more rapidly than the second-order perturbation energy. Consideration of these electron collisions in a metal suggests, therefore, that a modification of the wave function of electrons in a free electron model is required, which is not confined only to the surface of the Fermi sphere.

As the free electron theory may safely be applied to sodium (von der Lage and Bethe 1947), the discrepancy between theory and experiment found in § 3 is likely to be due to the one-electron approximation. Since we have not been able to find a new $2n$-electron wave function which ensures a more reasonable line width when all the terms of the perturbation U are used, the one-electron functions have been preserved and a screened potential of the form $r_{12}^{-1}\exp(-\lambda r_{12})$ has been introduced instead. It may be thought that the screening factor takes account of the fact that the Coulomb field due to one electron is screened off by induced charges at distances of the order of the mean distance between electrons in the metal. But this would be inconsistent as all electronic interactions have already been taken into account when calculating U_{if}, so that they cannot be invoked again at this stage. On the other hand, we have assumed, by adopting the one-electron approximation, that, apart from satisfying the Pauli Principle, the electrons move independently of each other, whereas in fact there are correlations for small distances, and our analysis suggests that there may also be some for larger distances.

For the new matrix element we obtain

$$U_{if}=4\pi e^2/V(g^2+\lambda^2) \text{ if } \mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_1'+\mathbf{k}_2' \text{ and } U_{if}=0 \text{ otherwise.}$$

This expression remains finite for small g and should be compared with (1). An argument analogous to, but more complex than, that of § 3 now yields a new and lengthy expresssion for $\Delta(k)$. If $\lambda\sim 1$ reciprocal Ångström unit, ϵ is found to be negligible, and one obtains

$$\begin{aligned}\Delta(k)=4{\cdot}31\Bigg\{&\left(\frac{k}{\lambda}-\frac{3\lambda}{4k}\right)\tan^{-1}\left(\frac{2k}{\lambda}\,\frac{k_F^2-k^2}{k_F^2+3k^2+\lambda^2}\right)\Bigg\}\\&+\frac{k_F^2-k^2}{2\lambda^2}\left[\frac{k_F^2-k^2}{2k\lambda}\tan^{-1}\frac{2k\lambda}{k_F^2-k^2+\lambda^2}-1\right]\\&+\log\frac{(k_F^2-k^2)^2+2\lambda^2(k_F^2+k^2)+\lambda^4}{4\lambda^2k^2+\lambda^4},\end{aligned}\qquad\ldots\ldots(9)$$

k and λ being expressed in the same units. Figures 2 and 3 give $\Delta(0)$ as a function of λ, and $\Delta(k)$ as a function of k. Skinner's results suggest that the maximum line width should be of the order of 1 ev., and hence Figure 2 shows that the value $\lambda\sim 1{\cdot}2$ A^{-1} should be suitable.

We now proceed, as explained in § 2, to replace each level by a broadened level. Thus the total probability per unit energy range that a level of energy E is occupied due to a broadened level at energy E_1, and of half-width $\Delta(E_1)$, is $[\Delta(E_1)/2\pi]\{(E-E_1)^2+[\Delta(E_1)/2]^2\}^{-1}$. The density of states with energies in the

interval E_1, E_1+dE_1 is, according to the free electron theory, $2\pi h^{-3}(2m)^{\frac{3}{2}}E_1^{\frac{1}{2}}\,dE_1$. Multiplying these two expressions, and integrating over E_1, one obtains the total number of electron states per unit volume of metal and per unit energy range, at the energy level E due to all broadened levels. Writing $\Delta(k_1)$ for $\Delta(E_1)$, and putting $E=\hbar^2k^2/2m$, we obtain the new density function

$$N(E)=\frac{1}{4\pi^3}\int_0^{k_F}\frac{\Delta(k_1)k_1^2\,dk_1}{[\hbar^2(k^2-k_1^2)/2m]^2+[\Delta(k_1)/2]^2}. \qquad \ldots\ldots(10)$$

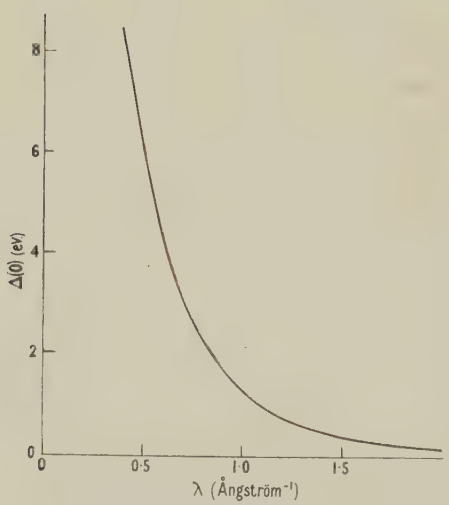

Figure 2. Graph of $\Delta(0)$ against λ using equation (9) with $k_F=0{\cdot}914$ A^{-1} (for sodium).

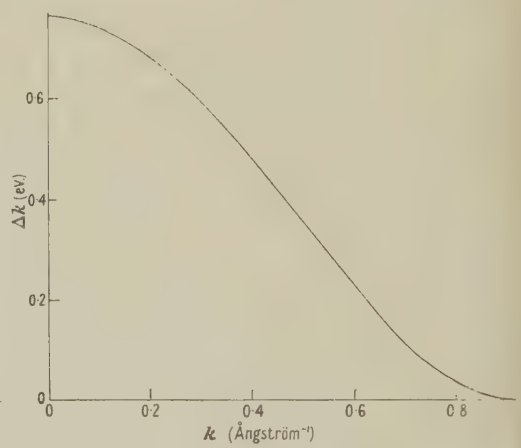

Figure 3. Graph of $\Delta(k)$ against k using equation (9) with $\lambda=1{\cdot}21$ A^{-1} and $k_F=0{\cdot}914$ A^{-1}.

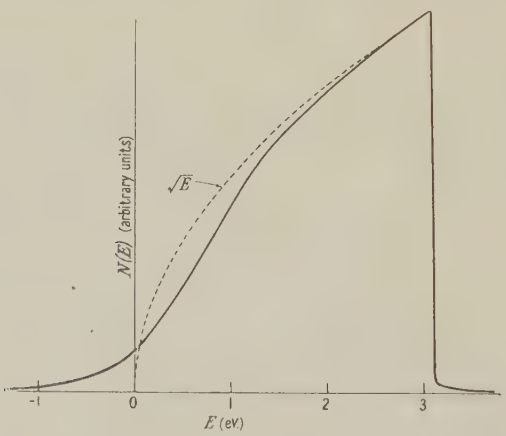

Figure 4. The $L_{II,III}$ emission curve of sodium according to the theory of §§ 2–4. Graph of the modified $I(E)$ curve using equation (10) with $\lambda=1{\cdot}21$ A^{-1} and $k_F=0{\cdot}914$ A^{-1}.

The values of $N(E)$ for various E were obtained by graphical integration, and are plotted in Figure 4. As the transition probability into a p-state is approximately independent of E in the free electron model, the $I(E)$ and $N(E)$ curves have the same shape in this approximation. It is seen that the present calculations (with $\lambda=1{\cdot}2$ A^{-1}) lead to a tail of the type found by experiment. Its length is, of course, governed by the value of λ which we have chosen.

§ 5. THE MORE EXACT ONE-ELECTRON WAVE FUNCTIONS BEFORE AND AFTER A RADIATIVE TRANSITION

We now turn to a calculation of the probability that an electron will make a transition from the conduction band into a vacant p-state. As explained in § 1, we wish to take into account that, prior to the transition being made, the potential in the neighbourhood of the ionized atom is not constant, but is depressed by an amount $D(r)$ which may roughly be represented by $D(r)=(e/r)\exp(-\mu r)$, at least for large values of r. One can obtain qualitative results much more readily by replacing $D(r)$ by a simpler potential. Accordingly, we shall suppose that the electrons are confined to a sphere of large radius R, the potential being zero between the radii a and R ($a \ll R$), and $-V$ up to the radius a. The position of the ionized atom determines the origin of the sphere.

To find the correct one-electron wave function in the two regions of space thus defined, one must solve two Schrödinger equations, whose angular parts lead to spherical harmonics. If l be the angular momentum quantum number, the radial equations have the general solution

$$R_l(r) = Fr^{-\frac{1}{2}}J_{l+\frac{1}{2}}(kr) + Gr^{-\frac{1}{2}}J_{-l-\frac{1}{2}}(kr),$$

with $k^2=\alpha^2\equiv 2m(E+V)/\hbar^2$ for the inner sphere and $k^2=\beta^2\equiv 2mE/\hbar^2$ for the outer region. E is here the energy of the electron, F and G are constants, and the J's are Bessel functions. The l-selection rule ensures that the main contribution to the total wave function comes from the radial terms for which $l=0$. Distinguishing the two regions of space by suffixes 1 and 2, and noting that ψ_1 must remain finite at $r=0$, one obtains for the radial wave functions

$$\left.\begin{aligned} \psi_1(r) &= Ar^{-1}\sin\alpha r, & 0 \leqslant r \leqslant a, \\ \psi_2(r) &= Br^{-1}\sin\beta r + Cr^{-1}\cos\beta r, & a \leqslant r \leqslant R, \end{aligned}\right\} \quad \ldots\ldots (11\,a)$$

A, B and C being constants. Since $\psi_2(R)=0$, one finds

$$\psi_2(r) = Dr^{-1}\sin\beta(R-r), \qquad D\equiv -B/\cos\beta R. \qquad \ldots\ldots (11\,b)$$

The continuity conditions at $r=a$ for ψ and its first derivative then yield the equations

$$\frac{A}{D} = \frac{\sin\beta(R-a)}{\sin\alpha a} = -\frac{\beta a\cos\beta(R-a)+\sin\beta(R-a)}{\alpha a\cos\alpha a-\sin\alpha a}, \qquad \ldots\ldots (12\,a)$$

and hence

$$\tan\beta(R-a) = -(\beta/\alpha)\tan\alpha a. \qquad \ldots\ldots (12\,b)$$

Since the left-side hand of this equation is a very much more rapidly oscillating function of E than is the right-hand side, (12) is satisfied only for the discrete set of energies E_j given approximately by

$$\beta_j R = j\pi, \qquad \ldots\ldots (13)$$

so that

$$\psi_{2j}(r) = Br^{-1}\sin\beta_j r, \qquad \ldots\ldots (11\,c)$$

and the appropriate density of states is

$$N(E) = (R/2\pi)(2m/\hbar^2 E)^{\frac{1}{2}}. \qquad \ldots\ldots (14)$$

In virtue of (12 *a*) and (12 *b*)

$$(A/B)^2 = \beta^2[\alpha^2\cos^2\alpha a + \beta^2\sin^2\alpha a]^{-1}, \qquad \ldots\ldots (15)$$

and the normalization condition gives approximately

$$|B|^2 = (2\pi R)^{-1}. \qquad \ldots\ldots (16)$$

The normalized wave functions are now obtainable from (11), (15) and (16). With $V=0$, they also give the wave functions of the electrons after the radiative transition has been made. It is evident from (13) that under these conditions the set of energy levels remains approximately unchanged, so that one can now specify the initial and final normalized one-electron wave functions, ψ_j and ϕ_j, thus:

$$\left.\begin{aligned}(2\pi R)^{\frac{1}{2}}\psi_j(r)&=\begin{cases}r^{-1}+(V/E_j)\cos^2\alpha_j a]^{-\frac{1}{2}}\sin\alpha_j r, & 0\leqslant r\leqslant a,\\ r^{-1}\sin\beta_j r, & a\leqslant r\leqslant R,\end{cases}\\ (2\pi R)^{\frac{1}{2}}\phi_j(r)&=r^{-1}\sin\beta_j r, \qquad 0\leqslant r\leqslant R.\end{aligned}\right\}\quad\ldots\ldots(17)$$

If there are $2n$ electrons, j goes from 1 to n, each state being occupied by two electrons with opposite spins, although the spin functions have not been given explicitly in (17). The angular functions have also been omitted. In the final state of the metal one energy level in the valence band is only singly-occupied, the other electron being in a p-state of the L shell, with normalized wave function χ (say).

§ 6. EVALUATION OF THE TRANSITION PROBABILITY

For a system of charged particles the probability of a radiative transition is proportional to $|M_{\mathrm{if}}|^2$, M_{if} being the matrix element of the total dipole moment $M=e\Sigma_k x_k$, the sum being over the x coordinates of all $2n$ electrons. The normalized initial and final wave function of the metal, Ψ_{i} and Ψ_{f} say, are the determinants

$$\Psi_{\mathrm{i}}=[(2n)!]^{-\frac{1}{2}}\Sigma_P\epsilon(P)P\psi_1(1)\ldots\psi_t(t)\ldots\psi_{2n}(2n),$$

$$\Psi_{\mathrm{f}}=[(2n)!]^{-\frac{1}{2}}\Sigma_Q\epsilon(Q)Q\phi_1(1)\ldots\chi(t)\ldots\phi_{2n}(2n).$$

The normalizing factors are correct since for large R

$$\int\psi_j^*\psi_k\,d\tau=\int\phi_j^*\phi_k\,d\tau=\delta_{jk};\qquad\int\phi_j^*\chi\,d\tau=0,\qquad\int|\chi|^2d\tau=1.\quad\ldots\ldots(18a)$$

Note also that

$$\int\psi_j^*\phi_k\,d\tau=\delta_{jk},\qquad\int\psi_j^*\chi\,d\tau=0.\qquad\ldots\ldots(18b)$$

The numbers in brackets label the electrons, and it is assumed that the tth electron has been made the transition into the L shell. The summations are over all $(2n)!$ permutation operators P and Q, and $\epsilon(P)$ is $+1$ or -1, depending on whether P is an even or an odd permutation. In virtue of (18) the matrix element reduces to

$$M_{\mathrm{if}}=A\int\psi_t^*x_t\chi\,d\tau_t/\int\psi_t^*\phi_t\,d\tau_t,$$

where A is independent of the energy E_t of the tth electron in the initial state of the metal. For large R the denominator is approximately unity, and since χ is a highly localized function, we can replace ψ_t by its value at $r=0$ without appreciable error. We then obtain for the transition probability as a function of E_t

$$P\propto E_t(E_t+V)[E_t+V\cos^2\alpha_t a]^{-1}.\qquad\ldots\ldots(19)$$

Using (14), the intensity is given by

$$I=C\sqrt{E'}(E'+V)[E'+V\cos^2\alpha' a]^{-1},\qquad\ldots\ldots(20)$$

where C is a constant and E' has been written* for E_t. I goes over into the simple theory, $I = C\sqrt{E'}$, if $V = 0$, and predicts a steeper rise than the simple theory; this is illustrated by Figure 5, in which some functions (20) are plotted. The

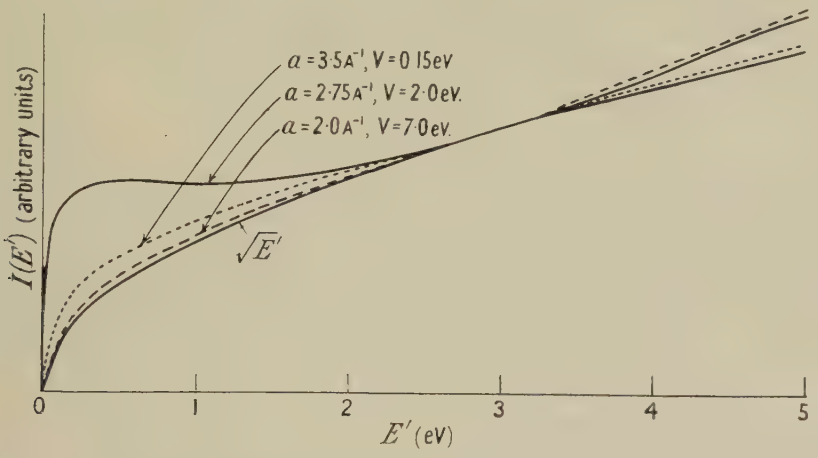

Figure 5. The $L_{II, III}$ emission curves of sodium according to the theory of §§ 5–6. The modified expression (19) for the transition probability, but not for the density of states, is used here.

values of a and V chosen ensure that the curves touch the function $C\sqrt{E'}$ at about 2·9 ev. The figures show that, although the screening effect must be taken into account, its treatment on the above lines is unlikely to improve agreement with existing experimental evidence. When the results of Figures 4 and 5 are combined, however, it is found that the effect of the screening is almost completely masked by the effect of the electron collisions in the conduction band.

§7. CALCULATION OF THE FINAL INTENSITY CURVE, TAKING INTO ACCOUNT BOTH PHYSICAL EFFECTS

The transition of free electrons into a p-state may be treated by using plane or spherical waves, which will be distinguished by suffixes p and s respectively. If P be the transition probability and N the density of states, it is well known that

$$P_p(E) \propto 1, \qquad N_p(E) \propto E^{\frac{1}{2}}.$$

On the other hand, reconsideration of §§ 5 and 6 without a potential well ($\alpha = \beta$) shows that

$$P_s(E) \propto E, \qquad N_s(E) \propto E^{-\frac{1}{2}}.$$

The treatment of electron collisions and of the screening effect yielded modified functions $\overline{N}_p$ and $\overline{P}_s$ respectively. Since the resulting intensities must, of course, be independent of the method of treatment, we obtain the three equations

$$P_p\overline{N}_p = P_s\overline{N}_s, \qquad \overline{P}_pN_p = \overline{P}_sN_s, \qquad \overline{P}_p\overline{N}_p = \overline{P}_s\overline{N}_s = \overline{I},$$

$\overline{I}$ being the desired modified intensity curve. Hence

$$\overline{I} = E^{-1}\overline{P}_s\overline{N}_p.$$

* E' has been used in Figure 5 in order to distinguish this energy coordinate from that of Figure 4. This difference assists in avoiding confusion when the results of Figures 4 and 5 are combined (cf. § 7).

This shows how the modified curves must be combined. The energy coordinates E and E', of Figures 4 and 5 respectively, must, however, first be fixed relative to each other. Figure 4 shows that the calculated density of states becomes negligibly small for $E=-1{\cdot}4$ ev. Hence we chose $E'=E+1{\cdot}4$ ev. In Figure 6 we compare a plot of $\bar{I}$ with Skinner's experimental results. The agreement is satisfactory, and much better than was previously obtained.

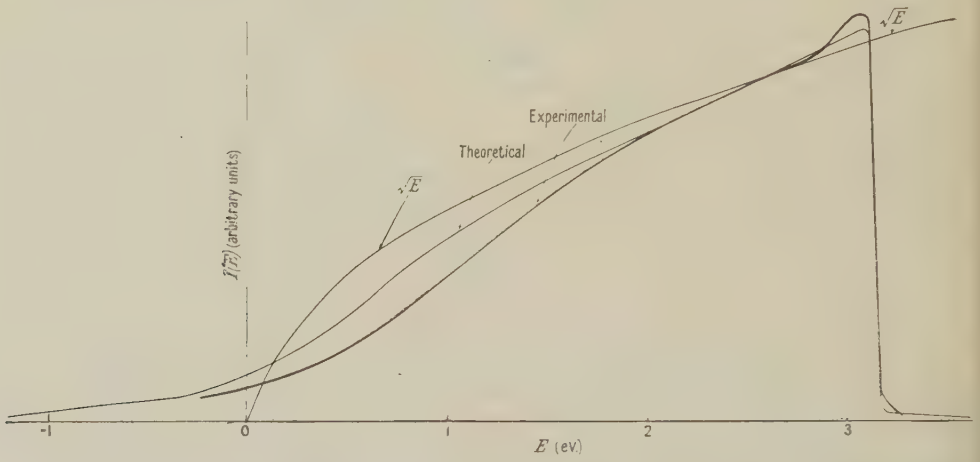

Figure 6. Experimental and theoretical $L_{II,III}$ emission curves of sodium. The new expressions for the transition probability and the density of states are combined in the theoretical curve, the constants being taken as $\lambda=1{\cdot}21$ A^{-1}, $k_F=0{\cdot}914$ A^{-1}, $a=2{\cdot}75$ A., $V=2{\cdot}0$ ev.

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The Luminescence Characteristics of Tin Activated Zinc Sulphide Phosphors

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ABSTRACT. It has been found that the inclusion of stannous compounds in relatively large concentrations in the preparation of zinc sulphide phosphors results in an intense red luminescence when excitation is by ultra-violet light of long wavelength (3650–4000 A.). Such characteristics as the luminescence spectra, excitation spectra, phosphorescence and thermoluminescence and the variation of luminescence with temperature have been studied. The hexagonal crystal form of zinc sulphide is essential to the production of efficient phosphors with tin as the activating impurity.

§1. INTRODUCTION

It has been found during the study of the effects of such metals as lead, tin and bismuth that tin can function as a luminescence activator in zinc sulphide phosphors. The reason for the absence of an earlier discovery of this activation must be ascribed to the previous use of small amounts of impurity, of the order of 0·1%, in preparation. In recent studies Fonda (1946) has shown that lead can function as a luminescence activator in zinc sulphide if relatively large amounts of lead salts are included when preparing the phosphors (he used 5% lead sulphate or nitrate in his preparations). In our experiments the amount of tin salt included in preparation, e.g. stannous chloride, was in some cases as high as 30% of the weight of zinc sulphide. Semi-quantitative chemical analysis and spectrographic tests indicated that only a very small proportion of tin remained after preparing the phosphor—of the same order as the activator concentrations in copper or silver activated phosphors (0·01%). The same spectrographic tests showed no presence of manganese or other significant impurities which might have given rise to the observed luminescence.

Our studies of tin activated phosphors included the following experimental investigations:

(i) Methods of preparing the phosphors.
(ii) Measurement of the spectral distribution of the luminescence for given excitation conditions.
(iii) Measurement of the excitation spectra of the phosphors—that is, the efficiency of radiation of different wavelengths in exciting luminescence.
(iv) Measurement of the variation of luminescence efficiency with temperature for the different spectral emission bands of the phosphors.
(v) Measurement of thermoluminescence characteristics which indicate the effect of tin activation on phosphorescence and electron trapping states in the phosphor.
(vi) Measurement of the characteristics of luminescence stimulated by infra-red irradiation after excitation of the phosphors by ultra-violet light.

We give the results of these studies and discussion of them in the appropriate sections below.

§ 2. PREPARATION OF TIN ACTIVATED ZINC SULPHIDE

The most suitable compounds of tin for inclusion in the preparation of tin activated zinc sulphide phosphors are stannous chloride and stannous sulphide: both must be of the purity standard of analytical reagents. Since both the chloride and the sulphide have low melting points compared with the temperatures of phosphor preparation, considerable amounts of the materials have to be added in preparation. In some cases, before heating, phosphor specimens contained as much as 30% by weight of stannous chloride. From results mentioned above we conclude that, after firing, the concentration of tin is only of the order of 0·01%. In order to obtain good phosphors it is necessary to use firing temperatures above 1,000° C. This is found to be so even when the tin is added to a zinc sulphide already preformed (i.e. crystallized) by firing at temperatures above 1,000° C. When phosphors are prepared at lower temperatures, with or without flux treatment, products are obtained which are almost non-luminescent in bulk. These contain specks of red-emitting material whose luminescence is characteristic of specimens made at higher temperatures. From this we conclude that the high temperatures are necessary for the diffusion of tin through the crystal lattice. All efficient phosphor specimens possess the hexagonal wurtzite structure, as shown by X-ray diffraction photographs. Phosphor formation is not appreciably influenced by inclusion of fluxes, such as sodium chloride, before firing. All our specimens were heated in silica tubes open to the air (tubes 12 in. long by $\frac{1}{2}$ in. internal bore, 3/64 in. wall thickness).

§ 3. THE LUMINESCENCE SPECTRA OF ZnS–Sn PHOSPHORS

The luminescence spectra of phosphor specimens were measured by means of a Hilger constant deviation spectrometer, photoelectric cell, electrometer triode system and pen recorder. The phosphor specimen was deposited in a thin layer on a glass slide and mounted in front of the entrance slit of the spectrograph. Excitation was by 3650 A. radiation from a 125 watt H.P. mercury arc lamp.

A typical emission spectrum is shown by the curves of Figure 1. Curve A is the corrected curve for equal energy while curve B is corrected to give equal quanta conditions. The spectra of all tin activated zinc sulphide phosphors are very much the same, consisting of a broad band in the blue spectral region characteristic of the self-activated phosphor and a broad band in the red region characteristic of the tin impurity. The maxima of these two bands are at 4700 A. and 6450 A. respectively. The relative intensity of the two bands is a function of the excitation intensity, the blue band increasing in intensity relative to the red band with increase in intensity of excitation, as in the case of ZnS–Mn (0·1%) phosphors (Gisolf and Kröger 1939).

The spectrum of Figure 1 was obtained at room temperature with a fairly high intensity of excitation. Red emission bands have been observed for zinc sulphide phosphors activated by iron (Kröger 1947), but the spectral maximum is then at 6100 A. and the emission intensity is always very much smaller than that which we observe for tin activation. As shown by Figure 1, the intensity of the red emission band is of the same order as that of the blue emission band and the latter is of the usual magnitude for other zinc sulphide phosphors (ZnS–Zn, ZnS–Cu (0·0001%), ZnS–Mn (0·1%)).

§ 4. EXCITATION SPECTRA OF ZnS–Sn PHOSPHORS

The excitation spectra of phosphor specimens were obtained by measuring the luminescence intensity produced by radiation of different wavelengths, the

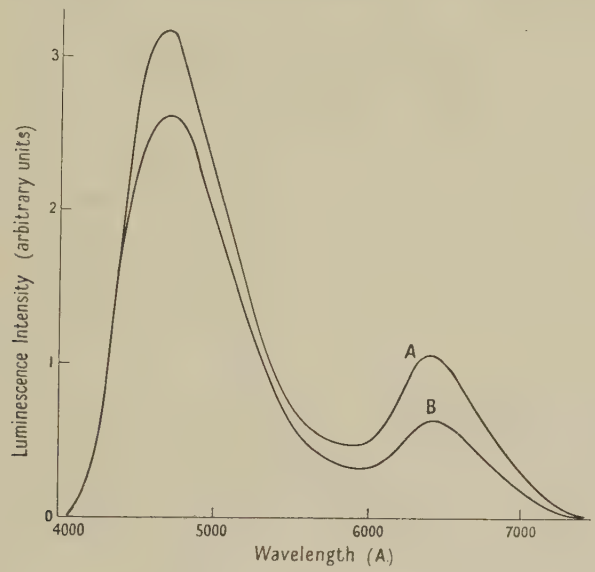

Figure 1. Luminescence emission spectra of a typical ZnS–Sn phosphor (λ=3650 A.).
A. Spectrum corrected to equal energy scale.
B. Spectrum corrected to equal number of quanta.

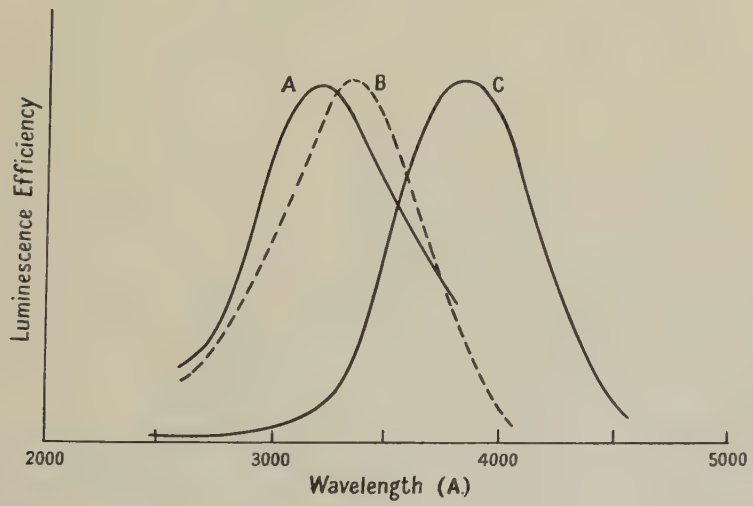

Figure 2. Excitation spectra for a typical ZnS–Sn phosphor at room temperature.
A. ZnS–Sn, blue emission characteristic of Zn activator.
B. ZnS, blue emission characteristic of Zn activator.
C. ZnS–Sn, red emission characteristic of Sn activator.

intensity of the latter being measured by a linear detector (in this case a thermocouple). A quartz prism monochromator provided the necessary monochromatic radiation. Excitation spectra for the blue and red emission of a ZnS–Sn specimen measured at room temperature are given in Figure 2. The spectrum for the blue

emission is similar to that measured for ZnS–Zn phosphors by previous workers (Garlick and Gibson 1948). The excitation spectrum for the red emission due to tin activation lies to longer wavelengths and has a maximum at about 3800 A. It is a broad band of somewhat similar form to that for the blue emission, but the excitation efficiency becomes negligible at wavelengths shorter than 3000 A. Excitation by ultra-violet light of short wavelength, by cathode rays or by X rays, produces a predominantly blue emission, the intensity of the red emission being very low. Since the absorption edge of the crystal lattice of zinc sulphide is at 3340 A., this means that excitation of the red emission (and thus of the tin impurity centres) is negligible when absorption takes place in the phosphor matrix. We hope to make further studies of the excitation spectra of these phosphor specimens at temperatures other than room temperature.

§ 5. THE VARIATION OF LUMINESCENCE EFFICIENCY WITH TEMPERATURE

The variation of the luminescence efficiency of tin activated zinc sulphide phosphors with temperature was measured separately for the blue and red emission bands, for constant excitation of fixed intensity and 3650 A. wavelength. Typical results are given in Figure 3. It is found that the blue emission efficiency decreases to zero in the same region (250–350° K.) as for self-activated zinc sulphide

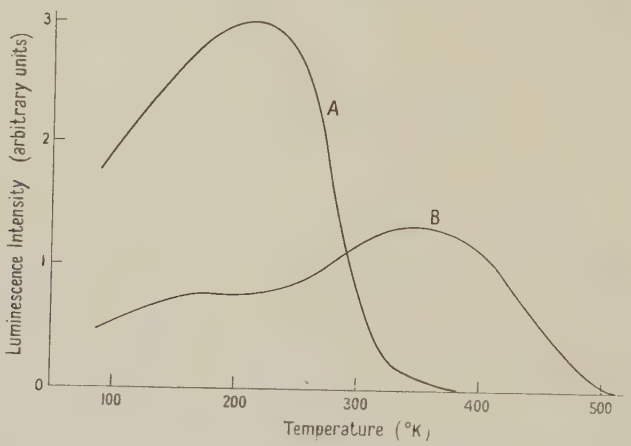

Figure 3. Variation of luminescence with temperature for a typical ZnS–Sn phosphor (constant excitation; $\lambda = 3650$ A.).
A. Blue emission characteristic of Zn activator.
B. Red emission characteristic of Sn activator.

phosphors (Garlick and Gibson 1948) but that the red emission persists to higher temperatures. As yet we have not studied the variation of luminescence efficiency with excitation intensity in the temperature regions 280–350° K. and 400–500° K. where the efficiency for the respective emission bands decreases rapidly with temperature due to non-radiative processes.

§ 6. THE THERMOLUMINESCENCE CHARACTERISTICS OF ZnS–Sn PHOSPHORS

To obtain the thermoluminescence characteristics of a ZnS–Sn specimen the phosphor is excited at 90° K. by 3650 A. radiation and then warmed at a uniform rate (2·5°/sec.) in the dark. The variation of its thermoluminescence emission with temperature is then recorded. Optical filters were used to obtain separately

the thermoluminescence curves for the blue and red emission bands. Typical results for three different specimens are given in Figure 4. The curves for the respective emission bands are very similar for all these specimens. The curves for

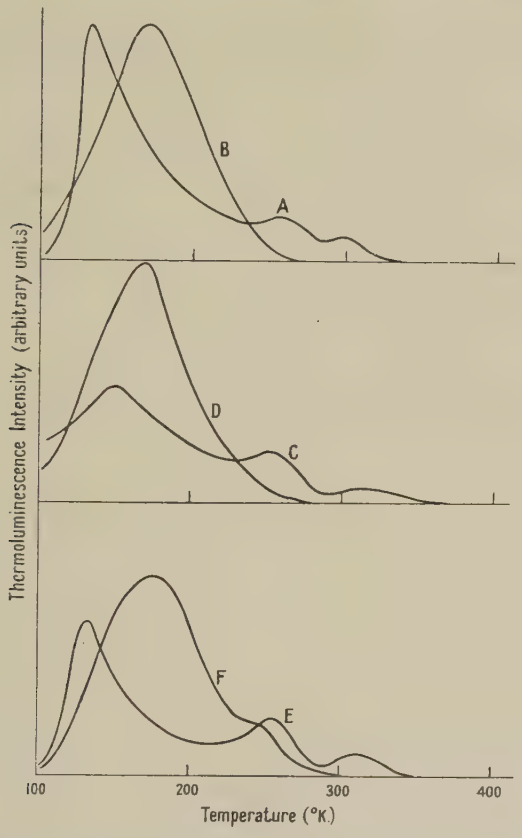

Figure 4. Variation of thermoluminescence intensity with temperature for different specimens of ZnS–Sn phosphor (excitation at 90° K.; λ=3650 A.).

A, C & E. Red emission characteristic of Sn activator.
B, D & F. Blue emission characteristic of Zn activator.

the blue emission are typical of other phosphors also (ZnS–Zn etc.), but those for the red band are characteristic of the tin activator. The peak at 130° K. is slightly variable since it corresponds to phosphorescence of short duration (seconds) at 90° K. The time elapsing between the removal of excitation and beginning of warming, which affects the peak position, was unavoidably different for each specimen in our experiments.

§7. THE INFRA-RED STIMULATION CHARACTERISTICS OF ZnS–Sn PHOSPHORS

Tin activated zinc sulphide phosphors have their blue emission band stimulated by suitable infra-red radiation at low temperatures (90° K.). The characteristics are very similar to those of pure zinc sulphide phosphors at these temperatures (Garlick and Mason 1949). The infra-red stimulation spectrum has a band at 1·2 μ strong compared with that of pure zinc sulphide phosphors. Stimulation does produce red emission in tin activated specimens, but this is too feeble to enable a measurement of the corresponding infra-red stimulation spectrum to be made.

§8. CONCLUSION

There appears to be no doubt that tin can function as an efficient activator for luminescence in zinc sulphide phosphors provided that the hexagonal crystal form of the phosphor is obtained by preparation at high temperatures. The tin impurity gives rise to phosphors which exhibit specific excitation and emission spectra, and gives rise to a characteristic distribution of electron trapping states as shown by thermoluminescence experiments. Preliminary experiments show that when cadmium sulphide is added to the phosphor in preparation its inclusion does not markedly influence the spectral position of the emission band due to tin impurity. We hope to extend investigations of this new type of phosphor to include measurements of absorption, excitation and emission spectra at various temperatures.

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ABSTRACTS FOR SECTION B

An Improved Method of Numerical Ray Tracing through Electron Lenses, by G. Liebmann.

ABSTRACT. An improved method is described for the tracing of electron trajectories through electron lenses by integrating the general ray equation with the help of Taylor's series, taking the series up to and including fourth-order terms. The paraxial trajectory is obtained by progressing from point P_n on the trajectory to point P_{n+1} with the help of recurrence formulae $r_{n+1}=Q_1r_n+Q_2r'_n$ and $r'_{n+1}=Q_3r'_n+Q_4r_n$, where the coefficients $Q_1 \ldots . Q_4$ are functions of the electrostatic or magnetic field distribution along the axis. The deviation of the first-order trajectory from the paraxial trajectory is built up with the help of similar recurrence formulae. The method is first developed for combined electrostatic and magnetic lenses, using the "equivalent potential" $U=\Phi-eA^2/2m$, and then specialized for pure magnetic or pure electrostatic lenses. The case of space charge within lenses is also considered. Comparison of the improved method with other methods shows that it is faster and capable of high accuracy. The factors influencing the accuracy are considered in detail and a short-cut method using extrapolation is given. A fully calculated example is appended.

Field Correctors for Parabolic Mirrors, by C. G. WYNNE.

ABSTRACT. Optical systems to correct the oblique aberrations of large parabolic mirrors in astronomical telescopes are discussed. The earlier work of F. E. Ross on doublet lens correctors in the converging beam is extended, and the residual first order spherical aberration is shown to depend only on the power of the corrector, its elimination requiring an impracticably large power; these results are independent of the arrangement and the optical constants of the components, and are unaffected by aspherizing the lens surfaces. Correction by two aspheric plates is shown to lead to systems of relatively great length, with consequent heavy vignetting. Finally, systems are discussed consisting of a doublet lens with one aspheric plate or secondary mirror; such a secondary mirror may be used to correct field curvature, giving corrector lenses which are afocal and therefore free from secondary spectrum, and the departure of the secondary mirror from the spherical form may be made zero by a suitable choice of the other variables. Details are given of the calculated performance of four three-component systems.

Shape Irregularities in the Equal Energy Luminosity Curve, by L. C. THOMSON.

ABSTRACT. The photopic luminosity function of two observers with normal colour vision has been measured. Two shoulders on the curve in the blue region of the spectrum first noticed by Gibson and Tyndall have been confirmed, and the relationship of the curves to the standard C.I.E. luminosity function is discussed.

The Acoustic Characteristics of Conical Pipes, by M. MOKHTAR and G. ABDEL MESSIH.

ABSTRACT. The paper describes an experimental investigation on slightly tapering conical pipes used as sound transmitters, undertaken in order to determine their acoustic characteristics, their end corrections and the pressure variation along the inside of the pipes.

A hot wire anemometer and a Pitot pressure tube were used for recording particle velocity and pressure variation in the acoustic field. The results obtained for the resonant frequencies show fair agreement with the theoretical formula $kr = -\tan kl$, where r is the radius of the throat of the conical pipe, l is its slant length and $k = \omega/c$, ω being the angular frequency and c the velocity of sound.

The recording of the acoustic pressure along the axis of the conical pipes shows that the nodes inside are places of minimum but not zero pressure, and that they are not equidistant. The end corrections are deduced from these pressure curves. The correction is always positive, being less than that of a cylindrical pipe of diameter equal to that of the mouth of the cone. It increases with the angle of the cone and with the order of the overtone to which the conical pipe resonates.

Measurements of the Velocity of Sound in Liquid Argon and Liquid Methane, by A. VAN ITTERBEEK and L. VERHAEGEN.

ABSTRACT. The ultrasonic velocity in liquid argon and methane has been measured between their boiling and melting points and the ratio of specific heats calculated. The compressibilities and degrees of association have also been calculated and compared with the results obtained for other liquified gases (hydrogen, helium).

The Behaviour of the Cathode Spot on an Undisturbed Mercury Surface, by K. D. FROOME.

ABSTRACT. The cathode spot of transient mercury arcs has been studied by means of a Kerr cell apparatus capable of taking many superimposed photographs. Discussion is limited to times when the spot is moving with sufficient velocity not to disturb the mercury surface. It is found that for a given current the " spot " takes the form of a line, or broken line, of total length proportional to the current. This is also true for a spot in a magnetic field. If the current rises slowly, the length of the line increases proportionally to the current; but if the rate of growth of current is greater than a value lying between $\pi \times 10^7$ and $2\pi \times 10^7$ amperes per second, then fresh spots form and spread out radially from the point of formation into lines moving with a maximum velocity of about 10^4 cm/sec. It is found that this velocity is never greatly exceeded whatever the experimental conditions. The current density is in excess of 10^6 amperes/cm^2.

Electron Flow in Curved Paths under Space-Charge Conditions, by B. MELTZER.

ABSTRACT. Further developments in a method of obtaining solutions of space-charge flow, described in a previous paper, are presented. A single example of three-dimensional flow is given, as well as a number of general theorems. A distinction is drawn between " normal " flow, in which all electrons have the same energy (example : a thermionic device with a unipotential cathode) and " abnormal " flow, in which they may have different energies. It is shown that in the case of the former the application of the general method simplifies considerably.

A Knife-Edge Balance for Weighings of the Highest Accuracy, by F. A. GOULD.

ABSTRACT. A detailed description is given of the balance used at the National Physical Laboratory for the most accurate weighings made in the comparison of fundamental standards of mass. Among the several unique features of this balance are the design of the beam, stirrups and pans, the use of recognized methods of geometrical constraint for precise location of these parts, the accuracy of construction, particularly of the knives, and the fineness of the positional adjustments of the components.

In technique of weighing, this balance also presents novel features. It is installed in a vault and, once it is loaded with the masses to be compared, the operations of controlling and reading the balance are entirely effected from outside the closed vault, mechanical means being employed for interchanging the loads undergoing comparison. Moreover, the technique of weighing is so arranged that a series of weighings of standards of mass from each arm of the balance in turn is completed without disturbing the contacts between the knives and their bearing planes. Largely as the result of using this technique, an accuracy of $\pm 0{\cdot}001$ mg., viz. 1 part in 10^9, has been attained in the comparison of kilogram masses of appropriate stability.

Onset of Turbulent Flow in Certain Arrays of Particles, by J. W. FOX.

ABSTRACT. Reynolds' number has been found for the onset of turbulence in water flowing through a perfectly packed hexagonal array of uniform spheres. Experiments have also been carried out on an imperfectly packed array of the same spheres and on an array of irregular particles (marble chips). A satisfactory differential manometer, which enables differences of head of the order of a hundredth of a millimetre of water to be readily determined, is described.

A Phenomenological Theory of Sintering, by J. K. MACKENZIE and R. SHUTTLEWORTH.

ABSTRACT. Sintering occurs when powders are heated to temperatures near their melting points. This paper deals with the rapid increase of density during the sintering of single substances. The increase of density cannot be explained by volume diffusion of vacant lattice sites or surface migration of atoms, but must involve macroscopic flow. The driving force for this flow is surface tension, and an equation connecting the rate of shear strain with the shear stress defines the resistance to deformation.

The density of a compact is calculated as a function of the time for two different laws of deformation, (*a*) that for a solid with a Newtonian viscosity, and (*b*) that for a Bingham solid which has a yield point and a rate of shear strain proportional to the difference between the applied shear stress and a yield stress. The effect of gas in the pores is calculated in the case of the viscous law.

The theory assumes that the pores are equal spheres and predicts that densification is uniform throughout a compact, independently of its shape and size, and suggests that gas pressures of a few atmospheres applied to the outside of a compact may appreciably increase the rate of sintering.

Relevant experiments and previous theories are examined critically, and it is shown that while the viscous model may explain the sintering of glasses it cannot explain that of metals. However, the experimental data can be explained by a model showing a yield point : on such a model the interaction of one pore with its neighbours is vital, so that pores in powder compacts close and isolated pores do not.

Obituary Notices

WILLIAM BOWEN

William Bowen, Managing Director of the Bowen Instrument Co. Leeds, died on 9th February 1949. He was born in September 1886, the eighth child and third son of John and Eliza Bowen. He had three brothers and seven sisters. His father was a coal miner; so was the eldest brother ($9\frac{1}{2}$ years older) and the second brother, David Bowen, K.C. ($1\frac{1}{2}$ years older).

He was educated at the Pentre Council Schools until the age of twelve. When he reached the school leaving age he wanted to become a coal miner but was persuaded to continue his education by entering the Porth County School; he remained there until, at the age of 17, he won a scholarship to the University College, Cardiff, where he took the courses in engineering. During that time (1904–5) the Welsh Revival movement occurred; with this Bowen was so taken up that he neglected his studies and lost his scholarship. His parents maintained him at College from 1905–6, but he did not succeed in getting his degree in engineering.

For a while after leaving college, in 1906, he taught at an elementary school in Swansea and then at a coaching school in London, after which he obtained a subordinate appointment with the Great Eastern Railway in the Engineers' Department. From there he entered the service of the Cambridge Instrument Company. Towards the end of 1913 he left this company to join his brother David Bowen as consulting engineers in Leeds. There he soon developed his own instrument company specializing in pyrometry, and up to the outbreak of the 1939 war his work was solely concerned with temperature measuring equipment.

In October 1939 he was asked to undertake the manufacture of compensating leads for aero engine temperature equipment. The insulation of these leads was of the plastics type and Bowen brought to bear his knowledge of temperature control equipment to secure a uniform coating of plastic on the cables. He then launched out into the production of plastic-covered wire and a vast amount was supplied in the form of assault cable for military purposes. He also produced a variety of plastic goods.

In the post war years he made strenuous efforts to popularize plastic-covered cables and read a paper on the subject before the Institution of Electrical Engineers. During this period he employed scientific assistance in development work. He contributed funds to the Scientific Instrument Manufacturers Association for the establishment of an award known as ' The Bowen Prize '. The resumption of peace time conditions and business competition imposed a severe strain which his constitution could not stand.

In 1946 Bowen and the writer made a B.I.O.S. trip to Germany and a warm friendship was formed as the result of it. He was a man of charming personality and was highly respected by his employees whose welfare was his first consideration.

In reviewing his life one realizes the profound influence that the higher educational system of Wales has had on the lives of the more ambitious members of the mining community, for without the opportunities given by scholarships and low fees at the University Bowen would probably have followed in the footsteps of his father and become a coal miner. It might be added that four of Bowen's family were students at the University College, Cardiff, during one session.

His brother, David Bowen, worked at the coal face as a miner for seven years then after four years at Cardiff College became the head of the Mining Department at the University of Leeds at the age of 25. David Bowen since the 1914–18 war had been at the Bar and was made K.C. in 1938. To him the writer of this obituary is indebted for details of Bowen's early life.

EZER GRIFFITHS.

NORMAN ROBERT CAMPBELL

In the death of Norman Campbell, which occurred on 18th May 1949, at the age of 69, the Physical Society has lost one of its outstanding Fellows. It is true that, because of ill-health, he had not been seen at meetings for ten years and more, but he continued to the very last to make characteristic contributions to the *Proceedings*, and these cannot fail to have reminded Fellows of the times when a paper by Campbell would fill the lecture theatre with an

audience that found his hard-hitting controversial attack on some respectable piece of orthodoxy most stimulating. He had a stammer, which must have been a severe handicap, but he certainly didn't let it stop him; and indeed it sometimes seemed to throw into stronger relief his burning enthusiasm for that particular facet of truth that was engaging his attention at the moment.

And here let it be noted that, though he could show an almost fanatical zeal in certain directions, he was no narrow specialist, but a man who delighted in science in all its aspects. Educated at Eton, and at Trinity College, Cambridge, he worked in the Cavendish Laboratory under the leadership of J. J. Thomson for several years, becoming a Fellow of Trinity College in 1904. He contributed the chapter 1903–9 to *The History of the Cavendish Laboratory*, and in so doing gave a good account of his interests at this period; ionization, radioactivity, penetrating radiation etc. He was appointed to the Cavendish Research Fellowship at Leeds at the time when W. H. Bragg was professor there, and continued on more or less the same lines of research, but he also wrote his *Modern Electrical Theory*, which had a great success, and made his name very familiar to students of that generation. He became an Honorary Fellow of Leeds University in 1913 and seemed well set for an academic career until the first world war brought a complete break by immersing him in applied physics.

Before this occurred, however, he had already shown signs of an interest that certainly forms no part of the Cavendish tradition; an interest in what might loosely be called the logic or philosophy of physics, though Campbell would probably have repudiated both terms. It first appears in a small book on *The Principles of Electricity*, which he wrote for the series of sixpenny books published by Jack, The People's Books. This must have puzzled many a novice. Electricity was presented to him not as that by which marvels have been accomplished, but that which provides wonderful opportunities for clear hard thinking about the first principles of science. Though Campbell subsequently followed many other lines of work, this was probably the one that he had most at heart, and he never abandoned it, although its reception seldom brought him anything but discouragement. In this vein we find his great book *Physics, The Elements* (1930) and the smaller ones *Measurement and Calculation* (1928) and *What is Science* ? (1921). All this work leaves the reader with no doubt that to Campbell the primary object of science was the enrichment of the human mind rather than the comfort of the human body. How strange then that he should have been content to spend the greater part of his working life in an industrial research laboratory ! The old term ' natural philosopher ' seemed to fit him so well that it was hard to see in him also the physicist as the handmaid of industry. But the fact is that he joined the staff of the Research Laborarory of the General Electric Company Ltd. on its foundation in 1919 and worked there until he retired in 1944. Moreover he obviously derived much satisfaction from his position as a member of this large and powerful research organization. This no doubt was partly because he had seen the organization grow up and had played an important part in establishing in it a genuinely scientific tradition, but it was probably partly also because he liked to have a hand in the ordinary workaday world. How else can one explain that after he had retired he voluntarily undertook to prepare a survey of systems of traffic lights, on finding out that one was needed.

As mentioned earlier, it was the War of 1914 that first deflected him towards applied science. He left Leeds for the National Physical Laboratory where he worked on magnetos and spark ignition with C. C. Paterson, work which after the war they published in the *Proceedings*. When Paterson left the N.P.L. in 1919 to organize research laboratories for the G.E.C. Campbell accompanied him and remained with him for the rest of his working life. The two men, so very different on a casual acquaintance, gave ample evidence that they valued the association highly. Paterson's original team indeed gave the impression of being a closely knit band of pioneers. One might be surprised to find Campbell in it, but, once in it was not difficult to understand his reluctance to drop out, and it was only ill-health that eventually forced him to do so.

He worked on the clean-up of gases by the electric discharge, took a leading part in the development of the caesium photoelectric cell, and wrote with Miss Ritchie a book on photoelectric cells. In this work we see Campbell as an experimental worker at the bench. With his tall lean form and intense concentration on the idea of the moment he could, when hard on the trail, be somewhat formidable. A laboratory assistant fresh from school admitted that she found him rather frightening, but for all that the impression that remains

is one of great personal charm. His working bench steadily accumulated a litter of discarded glass tubing, which it was absolutely forbidden to touch. When it reached a depth at which something simply had to be done, he himself would do it. Later on illness took its toll and he had to turn more and more to work on paper as distinct from work on the bench. He was to be found working on many problems in applied science; photometry, colour matching, statistics, ' noise ' in valve circuits, and even on the drafting of patent specifications. He confessed that this last, although quite interesting until he had mastered the technique, did become exceedingly boring, and probably nothing but the sense that it at least allowed him to keep his shoulder to the wheel, kept him at it.

All this time he maintained a passionate interest in the fundamentals of physics and produced from time to time the books that have already been mentioned and papers of a similar interest. He himself was convinced that his work on the roots of the subject was far more important than the endless pursuit of novelties, and that sooner or later the need for a far more critical attitude than is customary would be recognized, but he was under no illusion that physicists in general or even his own associates shared this opinion. There were undoubtedly times when he felt like a voice crying in the wilderness, but lest this should give a distorted impression of him it would be well to quote from an unpublished article he rote in the 1930's. He tells how on a day in 1913 a copy of the *Phil. Mag.* fell out of his bookcase and lay open on the floor.

> " Some algebraic formulae caught my eye .... It was part of a paper by a Mr. N. Bohr of whom I had never heard .... I sat down and began to read. In half and hour I was in a state of excitement and ecstasy, such as I have never experienced before or since in my scientific career. I had just finished a year's work revising a book on *Modern Electrical Theory*. These few pages made everything I had written entirely obsolete. That was a little annoying, no doubt; but the annoyance was nothing to the thrill of a new revelation, such as must have inspired Keats' most famous sonnet. And I had so nearly missed the joy of discovering this work for myself and rushing up to the laboratory to be the first to tell everyone else about it ! Twenty years have not damped my enthusiasm .... "

His friends will see the authentic Campbell in this fragment, and it is a good example of his writing at its best. One other example may be given; it is from a letter to the present writer, dated November 1944.

> " In March this year I carried into effect my long laid plans for retiring from all scientific work and adopting an unintellectual life in the country .... Our plans for a peaceful retirement were rudely shattered. After having endured the full length of the London blitz and having so many hairbreadth's escapes that we regarded ourselves as having charmed lives, we were caught by a stray bomb, jettisoned from a plane that was being chased, in an area that had never had a bomb before. Our house and almost all its contents were completely destroyed; I have lost all my papers and nearly all my books; and have had to take refuge here with some kind friends and to resume scientific work in order to give myself occupation. "

The quotation speaks for itself. It was followed by the suggestion that we should collaborate on a project that he had much at heart. The outcome can be found in the *Proceedings*, but it is Campbell's extraordinary fortitude in adversity that stands out most in these closing years. He and Mrs. Campbell faced the loss of their son on active service, the loss of their home and severe injury to Mrs. Campbell, but they could still enjoy Chaucer and he could work on the principles of electrical measurement. The death of Mrs. Campbell was a staggering blow, but once more he marshalled his forces, returned to the attack on the principles of magnetism, and looked round for a job (unpaid) in which he could help some scientific organization. It was in this way that he came to make a survey of traffic lights. Having settled down with his daughter near Nottingham, he came up to London for The Physical Society Exhibition of 1949, but the effort was too much for him and he died soon after his return, a man of science to the last.

It would have been pleasant to have had his photograph in the *Proceedings*, but it is characteristic of the man that none could be found.

L. H.

Prof. CECIL LEWIS FORTESCUE

Professor C. L. Fortescue's death on 22nd September, only three years after his retirement from the chair of electrical engineering at the Imperial College of Science and Technology, London, came as a shock to his many friends and former colleagues and staff. He was born in 1881 and educated at Oundle School and Christ's College, Cambridge, where he obtained a First Class in the Mechanical Sciences Tripos. He played an active part in school and college life, was a keen oarsman, and reached the ' trial eights ' while at Cambridge. This keen interest in student activities, and particularly rowing, never diminished, but probably increased during his later years, and he was rarely away from the river when Imperial College crews were racing.

On leaving Cambridge he spent some years at Messrs. Siemens' dynamo works and as instructor in electrotechnics and applied mechanics at H.M. Torpedo and Gunnery Schools, Portsmouth. During 1911–22 he was professor of electrical engineering at the Royal Naval College, Greenwich, and during this period, in addition to his academic and administrative duties, he carried out experimental work connected with wireless telegraphy during the first world war.

In 1922 he was appointed professor of electrical engineering at the Imperial College of Science and Technology, London. He was the third to occupy this position since 1885, the two previous occupants being Professors W. E. Ayrton and T. Mather. During the whole of the twenty-four years that he was professor and head of the Electrical Engineering Department he was indefatigable in his work for the College, and never seemed to begrudge the long hours spent in the Department. He was always accessible to the students and ready to help them in any way that was possible. He was, until his retirement in 1946, President of the College Radio Society, which involved giving twenty-four Presidential addresses, and he gave considerable time and energy to their preparation so as to present something which was interesting and instructive to the members of the Society. As a lecturer and teacher he was probably more successful with the advanced and postgraduate students than with those in the earlier years, who, in many cases, found his methods of dealing with electrical engineering somewhat difficult to appreciate. He will, generally speaking, be remembered at Imperial College for his very strenuous work in connection with the teaching of the subject of telecommunications at the undergraduate stage and the development of facilities for study and research in this subject for postgraduate students.

This development was so successful that it can be stated that the Electrical Engineering Department of the Imperial College provided a more thorough course of education in the subject of telecommunications than any other college or university in Great Britain, and students from all over the world were attracted to both the undergraduate and postgraduate courses.

As a professor of the University of London he took very seriously the many duties which fell to his lot, and devoted a great deal of care and attention to the work of the many committees and boards of which he was a member. He served on the Senate of the University, was Dean of the Faculty of Engineering and Chairman of the Board of Studies in Electrical Engineering.

In 1942 Prof. Fortescue was elected President of the Institution of Electrical Engineers, which was a fitting recognition not only of his professional standing and academic qualifications, but also of the devoted work he had done for the Institution for very many years.

Prof. Fortescue very rarely took any rest from his work, and since his retirement in 1946 he had been active in the service of the University and also at Southampton Technical College in connection with the development of courses in telecommunications, electronics and electrical measurements. He is survived by his wife, son and daughter.

A. R.

Dr. EDWARD PHILIP HARRISON

E. P. Harrison, born in London in 1877, died suddenly on 6th May, 1948. He was a Fellow of the Physical Society from 1900 to the time of his death. He was also a Fellow of the Institute of Physics and a Fellow of the Royal Society of Edinburgh.

Whilst a student in University College London he won an 1851 Exhibition Research Scholarship in physics and proceeded to the University of Zurich where he obtained his Ph.D.

In 1904 he went to King's College, Cambridge, and did two years research in physics at the Cavendish Laboratory. Later, he was appointed professor of physics in the Presidency College, Calcutta, where he was also head of the Observatory.

In 1923 he returned to England as Chief Scientist in the Mine Design Department, H.M.S. *Vernon*, Portsmouth, a research post under the (then) Director of Scientific Research, Admiralty. He remained in this post until 1937 when he retired on reaching the age limit. Much of his work for the Admiralty remained unpublished, on grounds of secrecy. His main interests however, centred around the application of magnetostriction to acoustics and magnetic measurements. He was the author of numerous original papers on these subjects. The principle of his ' A–E ' magnetometer, employing the change of electrical impedance of a mumetal wire in longitudinal magnetic fields, was used extensively in degaussing ranges during the war to measure the magnetic condition of ships.

On retiring from Admiralty service in 1937 he joined the research staff of Henry Hughes & Sons who at that time were concerned with the Admiralty magnetostriction echo depth sounder. Harrison's fundamental knowledge of magnetostriction helped considerably in the development of this device.

His pleasant personality and his persistence in the face of great difficulties will be remembered by his colleagues in the Admiralty.

Dr. Harrison is survived by his wife and one son. Another son, Robin, was lost in a ' Sunderland ' seaplane which failed to return to its base in the early days of the war.

A. B. WOOD.

RICHARD ALBERT HULL

The death of Dr. R. A. Hull, who was killed in a climbing accident on Mont Blanc on 22nd August 1949, robs Oxford of a distinguished experimental physicist and an outstanding successful college tutor. His friends will remember him as a man of plain-spoken sincerity and steadfast loyalty.

Richard Albert Hull was born on 27th March 1911 at Church Gresley, Derbyshire, and was educated at Ashby de la Zouch Grammar School. He entered Oxford in 1929 as an Exhibitioner of St. John's College, and read mathematics and physics, taking a first class in physics in the Final Examination in 1932. His first research was an experimental study of the photoelectric properties of transparent thin films of caesium and rubidium. The work was an exacting test of Hull's experimental skill, which was recognized when he was elected a Senior Scholar of Christ Church in 1934. He did not however find it an altogether satisfying field of work, and by the time he submitted his thesis for the degree of D.Phil. he had already turned to low temperature research. Hull took part in the investigation of the range of temperatures below 1° K. accessible by the new method of magnetic cooling. This was a field exactly suited to his temperament and gifts. Interested in experimental technique for its own sake, he enjoyed the exploration of the problems of adiabatic demagnetization, of temperature measurement, and of thermal insulation, which demanded solution before successful experiments could be made in the new field.

The conditions of work were often harassing and difficult. Experiments usually lasted from fifteen to twenty hours, and any accident or delay meant working throughout the night. Specific heat measurements in particular involved long hours of monotonous galvanometer reading. Hull was fortunate in a power of concentration which enabled him to keep command of an experiment in the most trying conditions. He worked himself hard, and the technique of work below 1° K. established in Oxford in 1939 owed much to his patient and ingenious experiments. He had published papers on the vapour pressure of liquid helium, on the attainment of low temperatures by evaporation of helium, and on magnetic effects in iron ammonium alum, one of the most important substances for the magnetic method. In addition, work had been completed on the properties of manganous ammonium sulphate, from which the absolute scale of temperature below 1° K. could be established by using this salt as a magnetic thermometer.

The war interrupted all this. Hull joined the research group formed at Oxford by the Admiralty to work on problems of radio waves of very short wavelengths, and began a study of millimetre waves. His outstanding contribution was the design and construction in 1941 of a velocity modulation oscillator for 9 mm. wavelength, which was used for investigations of propagation and reception of millimetre waves. This was probably the first continuous wave generator in the millimetre wave region.

Hull had been appointed a University Demonstrator and Lecturer in Physics in 1939, and as the war's demands increased he became more and more taken up with the teaching and training of physicists. He took a large part in the working at Oxford of the State Bursary scheme and the Hankey radio training scheme. In 1944 he spent a term as a visiting professor at Harvard, assisting in the U.S. Navy's radio training scheme there. In the same year he was elected a Fellow of Brasenose College. When undergraduates poured back to Oxford at the end of the war, Hull found himself the only science tutor in a large college, at least a quarter of whose undergraduates were reading science subjects. For a time he undertook the general care of all scientific studies in Brasenose, at the same time playing his part in the re-organization of the University physics department, where the number of students was treble the pre-war entry. In 1948 he became a member of the General Board of the Faculties, and soon became an accepted and vigorous representative of the science faculties. These activities made heavy inroads on the time he could give to research, but he returned to the field of work below 1° K. in which he had been engaged before the war, and at the time of his death was making experiments on the specific heat of liquid helium in this temperature range.

He was a rock of reliability in any matter entrusted to him. Any problem he took up, whether it was the troubles of a new pupil or the entrance requirements of the University, was considered long and deeply. When his mind was made up, he would speak it, even though it meant saying the difficult and unpopular thing. Something of this same deliberate quality was the secret of his success as a scientist and a teacher : never to accept a partial explanation and, when a satisfying answer was found, to reduce it to its clearest terms. He had many friends from a wide range of University life, from the hockey field, the Bach choir, and the Music Club, as well as those who enjoyed his company and quietly ironical humour in the closer society of common room. His nature however was perhaps too reserved for many to know him intimately. Accustomed to rely upon himself, he would seldom reveal his personal feelings or problems. Yet many brought their problems to him, and found him generous of time and trouble to help them.

He was an enthusiastic and very experienced mountaineer. Climbing was his passion; there can have been few vacations in the last ten years when he did not escape, at least for a short time, to the mountains. At the end of a hard term's work he would slip away to Scotland or Wales, or to the Alps, to return bronzed and refreshed, often with beautiful photographs of the rocks and mountains to delight his friends. He would persuade as many as he could to join him in the sport he loved, and was one of the leaders of this year's Alpine Club party to Switzerland. The climb on which he lost his life was to have been the last of the season—an ascent of Mont Blanc by the Brouillard ridge.

In 1937 Hull married Miss Judith Moore, like himself a research worker in the Clarendon Laboratory. She died in 1943. They leave one daughter.

A. H. C.

C. N. H. LOCK

Christopher Noel Hunter Lock was born on 21st December 1894. He was a Scholar at Charterhouse School and in 1912 was awarded a Major Scholarship at Gonville and Caius College, Cambridge. He became a Wrangler in Part II of the Mathematical Tripos, with distinction in Schedule B, in 1917. He was awarded a Smith's Prize in 1919 for an Essay on External Ballistics and in the following year was elected a Fellow of Gonville and Caius College.

During the 1914–18 war, and for two years afterwards, Lock worked on problems of external ballistics at Whale Island, Portsmouth, and at the National Physical Laboratory, Teddington. The results of an elaborate series of experiments on projectiles undertaken, in collaboration with R. H. Fowler, E. G. Gallop and H. W. Richmond, during the course of this work are given in two classical papers on " The Aerodynamics of a Spinning Shell ", published in *Phil. Trans.* A, Vols. 221 and 222. In 1921 he wrote, with R. H. Fowler, a paper on " The Origin of the Disturbances in the Initial Motion of a Shell " (*Proc. Camb. Phil. Soc.*, Vol. 20) and in 1922, also with R. H. Fowler, a paper on " Approximate Solutions of Linear Differential Equations " (*Proc. Lond. Math. Soc.*, Vol. 20).

In 1920 Lock was appointed to the staff of the Aerodynamics Division at the National Physical Laboratory. For about two years he worked on general aerodynamic problems such as the cushioning effect of the ground on aircraft performance and the lateral control

of aircraft, and then began his well-known work on airscrew problems, a subject on which he became an outstanding authority. He carried out both theoretical and experimental researches which contributed much to the development of modern airscrew theory and design. His researches, most of them described in reports published by the Aeronautical Research Committee, cover a wide field and include experiments with a family of airscrews, interference between airscrews and bodies, experimental verification of the independence of the elements of an airscrew blade, a consideration of the accuracy of the vortex theory of an airscrew, an application of Goldstein's theory to airscrew design, the flow around an airscrew in the vortex-ring state, and a graphical method for the calculation of airscrew performance. He also made wind-tunnel experiments on a model autogyro and contributed to the development of autogyro theory.

From 1939 until his death Lock organized and supervised the work undertaken in the High Speed Laboratory of the Aerodynamics Division. He played a prominent part in the design of the high-speed wind tunnels and in the development of the apparatus needed for experimental research, and his wide knowledge of the theory of compressible fluid flow was put to good use in the solution of difficult research problems. He took a very active part in all the work undertaken in the High Speed Laboratory and gave unstinted help to all his colleagues. His individual investigations include development of the pitot-traverse method for the determination of profile drag at high speeds, estimation of tunnel-wall interference at compressibility speeds, calculation of the ideal drag due to a shock wave, and a consideration of the effect of sweepback on the performance of an aerofoil at high Mach numbers.

Lock was a Fellow of the Royal Aeronautical Society and a Fellow of the Physical Society. He was a member of the Power Plant Committee, the Helicopter Committee, the Fluid Motion Sub-Committee and the Engine Aerodynamics Sub-Committee of the Aeronautical Research Council. He gave two lectures to the Royal Aeronautical Society, the first on "Airscrew-Body Interference" in 1929, and the second on "Problems of High-Speed Flight as affected by Compressibility" in 1937.

Lock died on 27th March, 1949, after a short illness. He suffered for many years from asthma and the after-effects of infantile paralysis, but he bore these bodily disabilities with great fortitude.

He married Lilian Mary Gillman in 1924 and his two sons, Robert Christopher and John Michael, are research students at Gonville and Caius College.

A. FAGE.

WILLIAM BLAIR MORTON

Professor W. B. Morton, who died in his native city of Belfast on August 12th at the age of 81, was one of the many physicists of his generation whose formal training had been principally in mathematics. He attended Queen's College, Belfast—then part of the Royal University of Ireland—from 1886 to 1889, reading mathematics under John Purser, and natural philosophy under J. D. Everett, and St. John's College, Cambridge, from 1889 to 1892, when he was eighth wrangler. He was profoundly impressed by Purser's methods of teaching, and strove in later years with conspicuous success to make his own lectures clear and interesting. Morton followed Everett in the chair of natural philosophy at Belfast in 1897 after being his assistant from 1892. He retained the chair until he was retired in 1933 on reaching the age limit. On appointment, one of his first acts was to increase the number of lectures given to undergraduates. Shortly afterwards, with the help of J. Wylie, he extended the laboratory classes. A weekly physical colloquium was started in 1904, after a visit to Germany. The conversion of Queen's College in 1909 into the independent Queen's University of Belfast brought no immediate change beyond the alteration of his title to Professor of Physics, but funds became available in 1911 for the erection of a new physics laboratory. This was designed by Wylie, and completed in 1914. Morton had to meet considerable criticism of its size, which was then considered excessive by some colleagues, and for various periods part of the basement was surrendered to the University's works and engineering departments, and part of the tower to chemistry. Good provision was made in the building for experimental research, although Morton himself took little part in it, preferring to devote such time as he had free from lectures to original theoretical work, and the history of mathematics and physics. Similarly, whilst he was very interested in the undergraduate laboratories, the detailed running of these was

left to Wylie. Although he was author or joint author of a considerable number of papers, he refrained from publishing much which might well have gone into print, preferring to talk about it in colloquium or postgraduate lectures. The present writer and others who were in Belfast in the late 1920's have much reason to be grateful to him for his expositions of quantum theory, which he recognized as being largely a transcription into a new context of the classical work of Rayleigh with which he was intimately acquainted.

Morton was a Fellow of the Royal University of Ireland from 1897 until its dissolution in 1909, a Member and for a time Vice-President of the Royal Irish Academy, and a Founder Fellow of the Institute of Physics. After his retirement, Queen's University conferred the degree of Doctor of Science on him, and he was made a member of Senate, its supreme governing body. He did not, however, seek for honours or public recognition, and was probably most happy in his work when he was in his department with his staff and students. To those who had the privilege of collaborating with him in any capacity his death has been a personal loss.

K. G. E.

ALAN FARADAY CAMPBELL POLLARD

THE death, on 15th August 1948, of Professor Alan Faraday Campbell Pollard deprived the world of science of one of the few who behold with increasing disquiet the spectacle of a community which spends money lavishly on research and the building up of knowledge, but gives little thought as to the ways in which this knowledge can be made accessible. Some of his favourite examples included the neglect, for 35 years, of the work of Mendel; the patent (1876) of Penaud and Gauchot which described many of the features of the modern aeroplane 27 years before the work of the Wright brothers; and the 81 years between the discovery of ' Herapathite ' and the perfection of Polaroid by E. H. Land. Pollard was an enthusiastic advocate of the Universal Decimal Classification and was the first President of the British Society for International Bibliography. For many years he kept up to date a U.D.C. catalogue of the *Proceedings of the Physical Society*. He served on the Council of the Physical Society from 1933–37.

Born in 1878, the son of Lieut.-Colonel B. H. Pollard of the Indian Staff Corps, he studied engineering at University College, London, and was for some time assistant to Professor Karl Pearson, who had a great influence in strengthening his passion for clarity and precision, and who gave him a bent towards the scientific study of instruments. After a period with Nobels Explosives Ltd. and war service (1914–18) with the Royal Scots Fusiliers, the R.A.F., and the Ministry of Munitions, he was appointed Professor of Instrument Design in the Technical Optics Department at the Imperial College and held the last-named post until his retirement from the College in 1943. He continued, however, to work as consultant and scientific adviser to various industrial undertakings until the time of his death.

He was very critical of the usual industrial approach to the design of scientific instruments and based his own ideas on the kinematical theories of Clerk Maxwell, which (used in original ways) led him to ingenious designs of various instruments such as interchangeable nosepieces for microscope objectives and other things. He inspired many students with something of his own enthusiasm, and explained some of his ideas in a small book, *The Kinematical Design of Couplings in Instrument Mechanism*, published by Messrs. Adam Hilger Ltd.

Many will regret that the Chair of Instrument Design has not been filled since his death. The design of instruments as a specific field of study has seemed to some inadmissible, the brewer preferring to design his colorimeter and the surveyor his theodolite. Yet if really adequate instruments are to be made there are fundamental principles of mechanical and optical design the neglect of which makes for failure instead of success. At a time when the scientific instrument industry of this country is making efforts to hold its own against strenuous foreign competition, our instrument industry may yet regret that Pollard's missionary efforts in respect of instrument design have not been followed up.

He was a friendly and helpful colleague; though he deplored the unsystematic and happy-go-lucky methods of some of us, he was always ready to listen and do his best to help. He married Gabrielle, daughter of Mr. Frederick Urwick, in 1915, and had a son and a daughter.

L. C. MARTIN.

WILLIAM GEORGE PYE

WILLIAM G. PYE, born on 27th October 1869, at Battersea, died at Bexhill on 13th October 1949. He was a son of William Thomas Pye, an expert maker of mechanical models. About 1878, A. G. Dew-Smith began making physiological instruments in Cambridge. In January 1881 Horace Darwin joined him as partner, to form the Cambridge Scientific Instrument Co. In December 1880 they offered W. T. Pye the post of foreman in a workshop they were to start on 1st January 1881. After a trial he was, in 1882, fully established as foreman. In 1895 the Directors of the then newly incorporated company made him Secretary: he had been Manager for some years. The company moved to St. Tibb's Row in 1881. When W. T. Pye came to Cambridge, from Battersea, his son attended the Higher Grade School. Then for a short time he taught at the Chesterton Church School and later did instrument work under his father. When W. T. Pye left, about December 1898, Mr. R. S. Whipple succeeded him as Secretary.

About 1886, W. G. Pye, now a trained youth, went to Elliott Bros., London. In 1892 the post of mechanic at the Cavendish Laboratory became vacant. At the opening of W. G. Pye's works at Chesterton (see below), Sir J. J. Thomson said :—

> " It was Mr. Dew-Smith who spoke to him about Mr. W. G. Pye, when they were seeking a new head of the workshop; he had every reason to be grateful for his advice to get Mr. Pye, if he could. He did get him, and they in the Cavendish Laboratory owed a great deal to him. He immensely improved the workshops and made a great many exceedingly effective instruments, and not the least he did was to train his successor, Mr. F. J. Lincoln. Mr. Pye, in addition to his great skill as a workman, possessed the qualities of great energy and business capacity, and so he encouraged him to start as an instrument maker."

I began to teach at the ' Cav. Lab.' in 1888. As a boy I did a lot of mechanical work. I naturally appreciated Pye's ' new broom ' and cordially echo the words of ' J. J.' There was leeway to make up. Maxwell made no provision for a workshop; he discovered his mistake and made a small beginning. When Pye came, in September 1891, the equipment was still meagre.

By 1892 the number of students had grown and class apparatus was needed. Pye made many things for the Medical Students—all very simple. Great additions for my own Class and for the Advanced Class were made by him or, under his direction, by Lincoln, who started as a ' lab. boy ' in 1891. With his first-rate mechanical knowledge, instinct for good design, handiness and skill of hand, Pye was very helpful. Research was developing and he made much of what the researchers could not ' pinch ' from the Practical Classes or make of ' sealing wax and string '.

Many research students, on leaving the Laboratory, wanted apparatus like that used there. To meet this demand, W. G. Pye, about 1895, set up a small workshop in St. Andrew's Street, with his brother, Frederick W. Pye, as foreman. On leaving the ' Cav. Lab.', in 1902, he equipped a workshop in a granary of a disused water-mill. He prospered and, in 1913, built works at Chesterton. In 1921 these works, which had grown, were attacked by the ' Radio Beetle ' and a rapid expansion followed. In 1927 the works and the radio business were bought by a company, under the name of Pye Radio Ltd., now Pye Ltd.

Pye and his younger son, H. J. Pye, then built the " Granta " works in Newmarket Road. Here the manufacture of electrical and mechanical apparatus for Practical Classes was continued by W. G. Pye & Co. Ltd. Mr. W. G. Pye soon became a ' sleeping partner '; the business was then run by H. J. Pye. In 1930 W. G. Pye left Cambridge and later went to Bexhill. There he took a leading part in the " Senlac Metal Casements Co. " till early in 1949. In 1946 W. G. Pye & Co. Ltd. was amalgamated with Pye Ltd.*

Many things devised for my Class were put on the market, often in improved forms, by Pye, and had a wide distribution and appeared in textbooks. The Conductivity of Copper apparatus stimulated a girl to write : " Conductivity for Heat, discovered by Dr. Searle ". When we were settling a design, a flashing suggestion from Pye sometimes altered the whole plan. I knew the destructive ways of students and demonstrators and Pye loathed things which ' came ungummed '. Sturdy and reliable apparatus was the result.

It is on record that I was pleased when, in 1911, I saw six sets of apparatus for the Mechanical Equivalent of Heat at Harvard University.

* Up to this point some of the dates given above may be only approximate.

From 1892 I had the friendship of W. G. Pye, and later of his son. We used to seek each other's help. They showed me much kindness in my work in Physics.

Sir J. E. Townsend, F.R.S., who began research in 1895, testifies :—

> " I am sure that the Research Students who worked in the Cavendish when Pye was head instrument maker feel that they are much indebted to him. I certainly appreciated very much the assistance he was to me. The cheerful way he undertook to make apparatus of various designs contributed substantially to the success of the research work."

C. T. R. Wilson, F.R.S., writes :—

> " I just remember him as being always most kind and helpful in his diffiultc task of supplying one's requirements from the Laboratory workshop."

In 1910 Pye went for his health to Torquay. With my wife and me he paid two or three visits to Oliver Heaviside.

I could not be at the opening of the Chesterton works in 1913. I wrote to Pye :—

> " You have the happy power of making a feeling of sunshine around you. This will go far to secure in the future, as it has done in the past, the loyal and enthusiastic co-operation of all those who help you in any way."

In July 1892 he married Miss Annie E. Atkins, who survives him. The surviving children are : (1) Donald Walter, of New College, Oxford, now Senior Master at Llandovery College, (2) Marjorie Irene, and (3) Harold John, of St. John's College, Cambridge, who attended my Practical Class.

G. F. C. SEARLE.

MAX PLANCK *

UNDOUBTEDLY Planck, who died at Göttingen on 4th October 1947, was one of the greatest of all those who ever turned their minds to scientific inquiry. His achievements place him among the three or four most outstanding men of science of this or any other time and, quite apart from his greatness as a contributor to our knowledge and comprehension of physical phenomena, he was a man of perfect integrity and great nobility of character, the like of whom rarely appears on our horizon.

Max Karl Ernst Ludwig Planck was born on the 23rd April 1858 at Kiel, where his father was a professor in the faculty of law. He was taken in his early years to Munich and was a pupil at the Maximilian Gymnasium for four years. In his recently published *Wissenschaftliche Selbstbiographie* † he remembers with affectionate appreciation a mathematical teacher there : one Hermann Müller, who was evidently a strong contributory influence in inclining him to take up the study of physics.

With the *abitur* of the Gymnasium Planck entered the University of Munich at the age of seventeen, where he studied under von Jolly, Ludwig Seidel and Gustav Bauer. After the manner of the German student, who was wont to migrate from one university to another, Planck spent a year at Berlin attending the lectures of Hermann von Helmholtz and Gustav Kirchhoff. He was promoted Doctor of Philosophy (*summa cum laude*) at Munich in 1879, his dissertation being entitled *Ueber den zweiten Hauptsatz der mechanischen Wärmetheorie*. Thereafter he became *Dozent*, the subject of his *Habilitationsschrift* being *Gleichgewichtszustände isotroper Körper in verschiedenen Temperaturen*. In 1885 he was called to the university of his native town, Kiel, as Professor *Extraordinarius*, having in the meantime completed the now well-known essay on the Principle of Conservation of Energy, which he submitted for the prize offered by the Göttingen Philosophical Faculty. Planck was adjudged second and one has a suspicion that the Faculty of that day were perhaps hardly capable of appreciating his essay at its full value.

In the spring of 1889 he was called to Berlin (as *extraordinarius*) to succeed Kirchhoff, on the recommendation, or initiative, of Helmholtz. He became Professor *Ordinarius* in 1892 and filled the office of Rector during the Semester of 1913–1914. His inaugural address was entitled *Neue Bahnen der physikalischen Erkenntnis*. In 1926 he retired and was succeeded by Erwin Schroedinger. He was awarded the Nobel Prize in 1918, taking as the subject of his address to the Royal Swedish Academy (on the 2nd June 1920) The

* This was received soon after the December 1948 issue of the *Proceedings* had gone to press.

† I am indebted to Professor Flint for providing me with a copy of this.

Origin and Development of the Quantum Theory. The Royal Society elected him a Foreign Member in 1926 and awarded him its Copley Medal in 1929. In 1930 he became President of the Kaiser Wilhelm Society. He also served for many years as one of the secretaries of the Prussian Academy, to which he was elected soon after the death of Hertz in 1894.

Planck's most important work was his investigation of the nature of black body (cavity) radiation, which led to the solution of the problem of the distribution of energy in the so-called normal spectrum and, as is well known, to the discovery of the quantum of action, h, and the quantum theory. Before his attention was directed to the radiation problem, he spent many years in the study of thermodynamics and in securing its foundations. No doubt this long thermodynamical training enabled him to appreciate better than anybody else, the nature of the formidable problem which he eventually solved.

His early work on thermodynamics evoked little response. Apparently it was not even read by Helmholtz, Kirchhoff or Clausius. This is perhaps not so surprising, when we reflect that thermodynamics must have impressed them as a finished product in which only trivial modifications could be contemplated. Indeed it still retains substantially the form which Clausius gave to it. It is perhaps easier to forgive this great triumvirate than later specialists in thermal science for their neglect of Planck. His attitude to thermodynamical principles was different from that of all his predecessors. He was not hypnotized by the literal sense of the terms ' reversible ' and ' irreversible '. He did not begin with reversibility at all, but by defining an irreversible process, which for him was a process which could not *in any way whatsoever* be completely undone (*rückgängig gemacht*). Any other processes were defined to be reversible. He seems to have been led to this attitude by studying the axiom of Clausius that heat cannot of itself pass from a colder to a warmer body. Notwithstanding Planck's criticisms this axiom is still used, quite uncritically, in quite recent textbooks.

Beyond doubt Planck succeeded in giving the second law a sound foundation, and his work, as set forth in his great book *Thermodynamik*, is rigorous; but there are alternatives to his approach, or at least one. One finds some indication of this in an unknown work by Carl Neumann (1875); but Neumann and Planck seem to have been unable to understand one another. They seem to me to be both equally sound.

In approaching the problem of the distribution of energy in the normal spectrum Planck made use of the fact, pointed out by Kirchhoff, that the character of the radiation within a vacuous enclosure, in temperature equilibrium, is independent of the nature of the emitting and absorbing material constituting the enclosing wall. He realized that this fact entitled him—in investigating the character of the radiation under temperature equilibrium—to assume *any sort* of emitting or absorbing systems he liked and so he assumed what seemed to him the simplest thing, namely the Hertzian oscillator. He had for his guidance the two formulae obtained by W. Wien (1896); in particular the exponential one which Lummer and Pringsheim had shown to fit the observations in the region of very short wavelengths. Probably at that time Rayleigh had not yet developed his formula; but its experimental equivalent had emerged from the work of Rubens and Kurlbaum in the region of very long waves—the residual radiation from rocksalt, fluorspar etc. Planck's preoccupation with thermodynamical studies, as he himself has indicated, made it natural that he should study the relation between the entropy of an oscillator and its energy. Wien's formula gives for U, the mean energy of an oscillator in temperature equilibrium,

$$U = ae^{-b/T},$$

a and b being constants. Now remembering that $dU/dS = T$, it easily follows that

$$1 \Big/ \frac{d^2S}{dU^2} = -bU,$$

S and T meaning entropy and temperature respectively. On the other hand the results of Rubens and Kurlbaum suggested $U = cT$, where c is a constant. This in effect is Rayleigh's formula. Consequently

$$1 \Big/ \frac{d^2S}{dU^2} = -\frac{U^2}{c}.$$

Naturally Planck tried the experiment of combining these two in the following simple way :

$$1 \Big/ \frac{d^2S}{dU^2} = -bU - \frac{U^2}{c},$$

the solution of which yields

$$U = bc/(e^{-b/T} - 1),$$

which is virtually Planck's final formula; but the problem of deducing it from fundamental principles still remained. In order to make progress in this direction Planck turned to Boltzmann's H theorem and its implication of a functional relationship between the entropy of an assemblage and the probability of its state. He was thus led to equate the entropy of an assemblage to the product of a universal constant and the logarithm of what he called the thermodynamical probability. This universal constant, which he represented by $\boldsymbol{k}$, is usually called Boltzmann's constant; but was in fact first introduced by Planck. Indeed as Planck has said, his theory is distinguished from that of Boltzmann precisely by the assumption or premiss that a definite finite number, $\boldsymbol{k}$, actually exists, with the property that its product with the logarithm of the thermodynamical probability is equal to the entropy.

The rest is very simply told. In any case, whether a departure from classical principles were contemplated or not—and probably Planck did not have this in mind at first—it was necessary to divide the extension in phase of the oscillators—each assumed to have one degree of freedom—into elements with the dimensions *energy* × *time*. The classical procedure necessarily involves that these elements approach the limit zero. Planck realized however that this gave the oscillators the average energy $U = cT$, which could only fit the observations in the region of very long waves. Thus he was forced to endow each element of the phase space with a precise numerical value, *different from zero*. He represented this element by the symbol, $\boldsymbol{h}$, and called it the *elementary quantum of action*. It was a consequence of this that the energy of each oscillator took the form

$$\text{Integer} \times \boldsymbol{h}\nu,$$

or, in his later and slightly modified theory—in which he assumed that *energy emission only* was discontinuous—

$$(\text{Integer} - \tfrac{1}{2}) \times \boldsymbol{h}\nu.$$

Just as the constant $\boldsymbol{k}$ was suggested to Planck by Boltzmann's theory, so the much earlier work of Sir William Hamilton might have suggested the constant $\boldsymbol{h}$. It appears later, in wave mechanics, not only as the quotient of energy by frequency, but also as the quotient of momentum by wave number. Indeed the study of Hamilton's analogy between geometrical optics and classical mechanics might well have suggested a more general mechanics analogous to optics or wave propagation in the widest sense.

Planck communicated his theory to the German Physical Society and it was published in their *Proceedings* and in the *Annalen der Physik* at the very close of the century. Anyone who can remember the state of physical science in 1900 and the outloook of physicists at that time will appreciate that Planck's theory of radiation was at first regarded as very eccentric if not quite mad; but when one re-reads it one realizes that it is an almost inevitable consequence of the actual facts of black-body radiation already known. A very strong impression was made by Planck's computation of Loschmidt's (or Avogadro's) number and consequently of the elementary ionic charge, which he found to be $4{\cdot}6 \times 910^{-10}$ E.S.U. Rutherford and Geiger found $4{\cdot}65 \times 10^{-10}$ E.S.U. a year or two later by counting alpha particles and estimating their total charge. Planck's result is remarkably close to $4{\cdot}803 \times 10^{-10}$ E.S.U., which I think is the present estimate. He computed $\boldsymbol{h}$ to be $6{\cdot}55 \times 10^{-27}$ ergs × sec., a value which has not been appreciably modified since, and $\boldsymbol{k}$ he found to be $1{\cdot}34 \times 10^{-16}$ ergs per °K., now estimated to be $1{\cdot}372 \times 10^{-16}$ ergs per °K. Many important consequences soon followed from Planck's theory of *Quanten-emission* even before a coherent quantum mechanical theory was built up: Einstein's photoelectric theory; the dependence of specific heats on temperature, especially Debye's brilliant treatment of the atomic heats of solid elements; Bohr's theory of the hydrogen atom and of the extra-nuclear structure of atoms and the theory of spectra; the contributions of Sommerfeld, Epstein, Kramers and others and then the triumph of quantum mechanics (L. de Broglie, E. Schroedinger, W. Heisenberg, M. Born and P. Dirac).

Planck was twice married and is survived by his second wife (Margaret, *née* von Hoesslin) and a son. Until the outbreak of the first world war he might indeed have been described as a happy and fortunate man, particularly fortunate in his teachers, in his academic career and in making one of the greatest of all scientific discoveries. But then calamities came. His eldest son, Karl, fell before Verdun in 1916 and he lost his twin daughters. In the last war his house in the Grunewald quarter of Berlin was destroyed in an air raid and his fine library lost. He happened to be in Cassel when that town was destroyed and was buried for hours in an air-raid shelter. After the attempt on Hitler's life, on 20th July 1944, his son Erwin was arrested, condemned and executed on 23rd Jan. 1945, solely on the ground, apparently, that some of those involved belonged to his circle of acquaintances.

Planck was found in May 1945 by the Americans at Rogätz on the Elbe and conveyed by them to Göttingen where he remained till his death and where his remains now lie.

Many Fellows of the Physical Society will remember his Guthrie lecture in 1932 and his profoundly interesting and illuminating discussion of causality. He had been elected an Honorary Fellow of the Physical Society in 1924. In 1936 he attended, as a guest, the centenary celebrations of the University of London, and it was on this occasion, I believe, that the University conferred on him the degree of Doctor of Science *honoris causa*. He obviously much enjoyed his visits to St. Paul's Cathedral and to the Guildhall and the dinner in his honour at the Waldorf Hotel.

His last visit to England was in 1946, little more than a year before his death, to attend the Royal Society's Newton celebrations. He was no longer the erect and vigorous man that many of us remember; but he retained his mental clarity and alertness.

Planck was an enthusiastic musician and a fine pianist. He was also a great mountaineer and continued to make strenuous ascents of 3000 metres or more after reaching 70 years of age. I have it on the authority of Professor Sommerfeld that Planck made personal representations to Hitler on behalf of persecuted Jewish scientists. He was of course '*schroff zurückgewiesen*'.

I remember him as a modest, kindly man, who, in the words of one of the greatest of his countrymen,

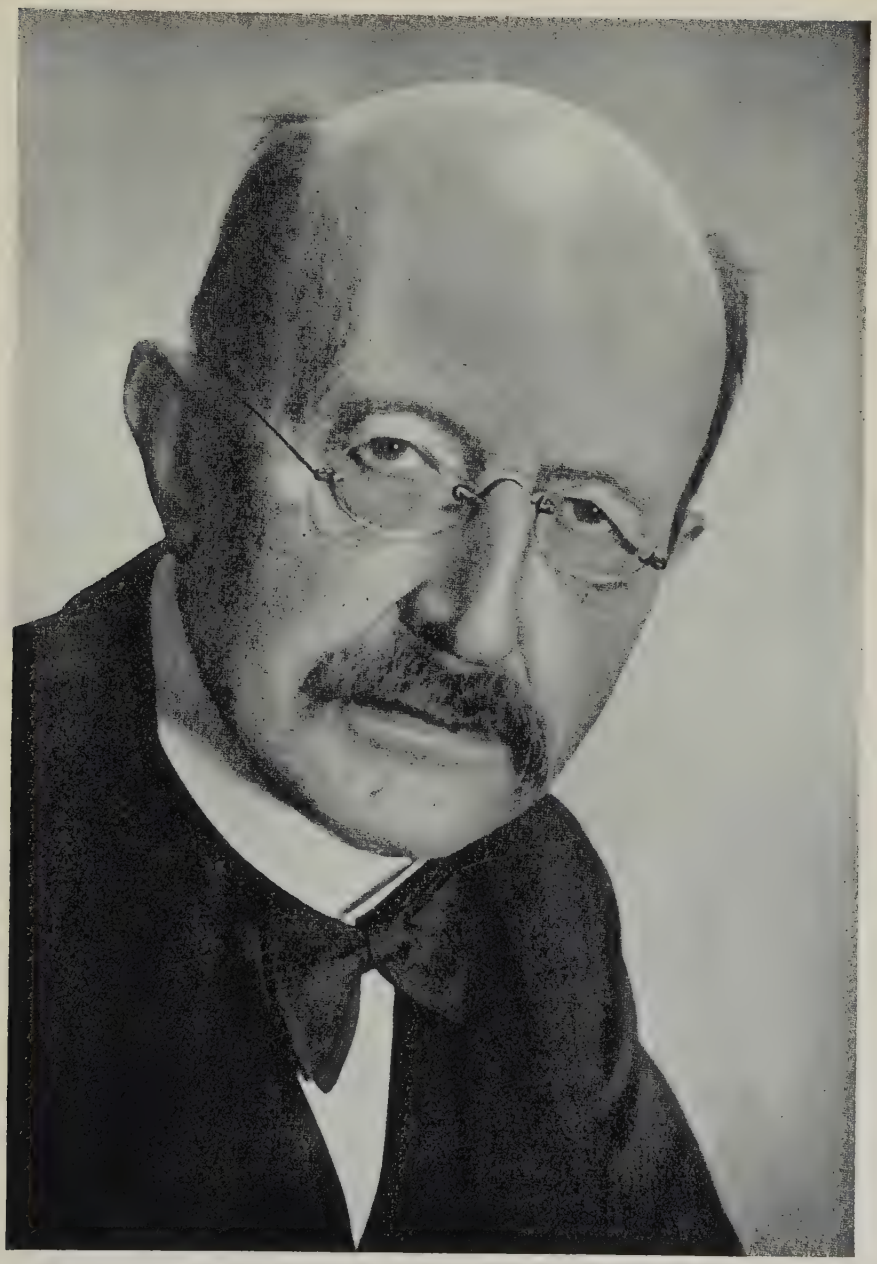
weit entfernt von allem Schein,
Nur in der Wesen Tiefe trachtet.

WM. WILSON.

WILLIAM NELSON STOCKER

THE name of W. N. Stocker, who died on 2nd August 1949 at Oxford, in his 99th year, will probably sound unfamiliar to most readers of the *Proceedings of the Physical Society*. Even in, Oxford where he taught physics on and off for over forty years, his name is better known to those interested in original 'Oxford characters' than to the physicists. And yet Stocker, who was born in the year Kelvin formulated the Second Law of Thermodynamics, who took Finals in the year Maxwell published his *Treatise*, and who, in his own words, 'lost the habit of work' in the year Einstein announced the Special Theory of Relativity, was for the last ten years or so the *doyen* of British physicists.

William Nelson Stocker was born on 29th January 1851 at Horsforth, near Leeds, the son of the Rev. William Henry Browell Stocker. After education at Stony Stratford School he matriculated at Oxford in 1869 with the intention of reading classics and taking Holy Orders. His interest in science soon caused him to change his plans, and in 1870 he started studying mathematics and physics, gaining a First in Mathematical Moderations in 1872 and two Firsts in the Final Honour Schools of Mathematics (1873) and Natural Science (1874). It was presumably for lack of sufficient money that he had started as a non-collegiate student and remained one until 1873, when he gained an Exhibition at Christ Church, and his first five years in Oxford must have been a period of hard work, modest living and recreations chosen with an eye on a tight budget. Little is known about these years, but from the detailed entries of his account book covering the years 1869–1878 one pictures young Stocker as an undergraduate who, within his modest means, enjoyed the amenities that Oxford could offer. Boating and skating alternated with long walks in Oxford and its neighbourhood, books and music were bought for study and for leisure; then there were public concerts (the Hallé Society's concerts seem to have been regular features in those



Dr. Max Planck.

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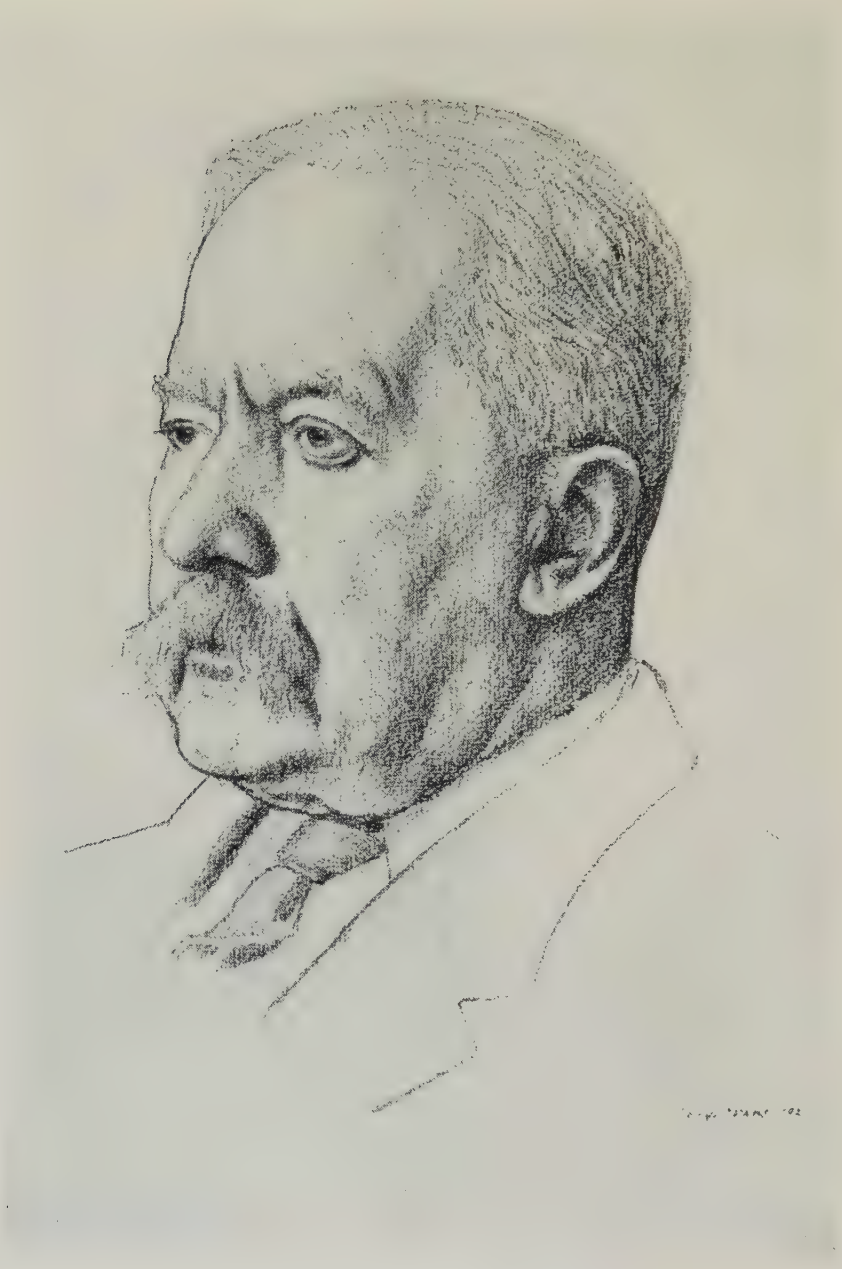

WILLIAM NELSON STOCKER

days) and the private enjoyment of playing the piano and the organ. We also learn from this account book that the cost of his fifteen terms at Oxford, including University expenses, board and lodging, books, etc. was just under £400, a very modest sum even for those days.

He took his degree in 1874, and in the same year accepted a demonstratorship at the then new Clarendon Laboratory under Professor R. B. Clifton in preference to becoming a science master in Cheltenham. In 1877 he was elected to a Fellowship at Brasenose for life, and his death marks the disappearance of the last of these Oxford life-Fellows. In the same year he became a Fellow of the Physical Society and, having held this Fellowship for 72 years, established a record which will be very hard indeed to break. His generous contribution to the Physical Society's Holweck Fund, one of his many donations to deserving causes, will be gratefully remembered. He was one of the first members of the Oxford University Mathematical and Physical Society, founded in 1888, and although there is no record of his ever reading a paper to that Society he took a lively interest in its functions, and he was genuinely disappointed when, in November 1948, failing health prevented him from attending the dinner commemorating the Society's 300th Meeting. Stocker was a conscientious if not inspiring teacher. He was more interested in experiment than in theory, and many generations of undergraduates who passed through his hands in the Clarendon Laboratory learned from him how to carry out physical experiments carefully, honestly and accurately.

In 1883, at the age of 32, Stocker was appointed Professor of Physics at the Royal Indian Civil Engineering College at Cooper's Hill, near Egham, Surrey. He held this post for the next 18 years till 1901 and delivered regular courses of lectures on heat, electricity and magnetism, light, etc. He used to visit Oxford for occasional week-ends during these years, keeping up his old connections with Brasenose, so that when in 1901, at the age of 50, he retired from Cooper's Hill and returned to Brasenose it was like coming home. He continued to do some teaching for the College and demonstrating in the Clarendon Laboratory, but these activities were on a diminishing scale, and came to an end soon after the 1914–18 war. One important event during this period was his " around the world trip ", as he called it. He went to Australia via the Indian Ocean in the summer of 1914 to attend the British Association Meeting there and returned via Canada, arriving back in England in October.

Little can be said about the last 30 years of Stocker's life, especially about his scientific pursuits. He was never much interested in new developments in physics and with ' Einstein and all that ' he did not even attempt to cope. He did, however, like to hear gossip about what was going on in the laboratories and about the doings of some of the old guard whom he still knew. A few years before his death he was shown round the new Clarendon Laboratory by the late Dr. R. A. Hull, his third successor as Physics Tutor at Brasenose. He seemed to have been impressed by what he saw in the new laboratory, but perhaps even more by the sight of the old Clarendon being partly pulled down and rebuilt. It must have given Stocker some satisfaction to see that he survived not only all his human contemporaries but even the Laboratory which he knew when it was first built.

If it is difficult to give an account of Stocker's life—partly because it was so uneventful—it is even more difficult to describe his personality without the fear of doing him an injustice. What we know about him is bound to be superficial; he had very few intimate friends—most of whom have been dead for a long time—and even to them he was never communicative. This does not mean he did not like company; in fact among his greatest pleasures were the long, friendly evenings in Brasenose Senior Common Room, and he had little patience with some of the younger generation who would ' use the place as a restaurant ' and go off after dinner to meetings or lectures or—*horribile dictu*—to the laboratory to continue experiments.

To those who met Stocker in the last one or two decades the mere fact of talking to a man really belonging to quite a different age was a thrilling experience. Stocker himself was rather proud of his old age and of his reminiscences going back well over 90 years. He could remember Tsar Nicholas I being burnt in effigy at Horsforth during the Crimean War; one of his ' earlier ' recollections was of Queen Victoria and the Prince Consort driving in state to the opening of the new Guildhall of Leeds, and he would delight in mentioning that his uncle, Dr. C. W. Stocker, Fellow of St. John's, had examined Gladstone in 1831. It is no wonder that after hearing such reminiscences one almost believed Stocker's classic

remark about lectures on electricity : " In my days you took a piece of sealing wax, rubbed it against your sleeve, picked up a piece of paper with it, and there was your course of electricity ".

By no means a brilliant conversationalist, Stocker had the gift of a dry, biting humour which enabled him to drive home a point with a minimum outlay of words and, very often, the maximum embarrassment to the unfortunate victim. Among the many stories told about him the following quite authentic one may be given as an example. It so happened that on VJ Day in 1945 a party of only four was dining in Common Room; Stocker, two people he very much disliked, and a fourth person who, the last to arrive, greeted Stocker with the remark, " A small but select gathering for this memorable day ! " After a brief pause came Stocker's grunting reply, " Yes . . . humph . . . small." In spite of this mordant wit he had an air of measured old-world courtesy, and many a young guest in the Common Room must have felt surprised and flattered at being addressed as " Sir " by an old gentleman more than 70 years his senior.

Stocker's great hobby—it was an integral part of his life—was walking. His regular daily walks at Oxford and Cooper's Hill were followed in the vacations by long walking tours during which he explored the countryside and observed the flora and geology of all parts of Britain. He always carried a pedometer and a barometer on him, and kept an exact record of the distances covered and the heights ascended. His endurance was fabulous. Even when he was over 70 he could average twenty miles a day over several weeks, and one can quite believe that in his life-time he all but covered the distance to the moon and did over a hundred ascents of Everest. These and similar data are contained in some seventy-five volumes of diaries which also present a faithful record of the weather from 1873–1949 with thermometric and barometric readings.

One might wonder why it was that Stocker, undoubtedly an able man with a capacity and a constitution for hard work and with a deep interest in nature, never made a name for himself in the world of physics. One should remember, however, that in 1874, when Stocker became a demonstrator, and for nearly 50 years afterwards, the Clarendon Laboratory was a place of teaching from which research was virtually banned. Whatever may have been Professor Clifton's merits as a teacher or as a collector of the choicest scientific instruments of his day, he certainly was not a man who could have inspired young Stocker to engage in research and to take up physics as his hobby. It is perhaps not idle to speculate whether, given a more inspiring scientific environment during his earlier years, Stocker might not have become the Grand Old Man of Oxford Physics. As it is, one thinks of him as an original and therefore typical old bachelor don of the Victorian era whose sayings, doings and habits, enriched and embellished by the passage of time, will probably be handed down as a legend to posterity.

N. KURTI.

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PROCEEDINGS AT THE MEETINGS OF THE PHYSICAL SOCIETY

SESSION 1948–49

13*th October* 1948

Joint Science Meeting of THE COLOUR AND OPTICAL GROUPS, at Northampton Polytechnic, London E.C.1. Mr. J. G. Holmes was in the Chair.

The following papers were read and discussed :

" General Phenomena associated with Scattering of Light ", by E. W. H. Selwyn;

" Atmospheric Haze Factors : An Approximate Calculation of the Diffuse Reflection and Transmission of a Layer of Scattering Particles ", by W. S. Stiles;

" The Effect of Haze on Photography from the Air at Night ", by G. B. Harrison;

" Scattering of Light in Photographic Materials ", by D. T. R. Dighton;

" The Scattering and Absorption of Light in Coloured Paper ", by F. North.

14*th October* 1948

The ninth meeting of THE ACOUSTICS GROUP, at Imperial College, London S.W.7. Mr. H. L. Kirke was in the Chair.

The following paper was read and discussed :

" The Accuracy of Measurements by Rayleigh Disc ", by W. West.

25*th October* 1948

The sixteenth meeting of THE LOW TEMPERATURE GROUP, at the Science Museum, London S.W.7. Sir Charles Darwin was in the Chair.

The following paper was read and discussed :

" The Unattainability of Absolute Zero ", by F. E. Simon.

4*th November* 1948

The tenth meeting of THE ACOUSTICS GROUP, at the Royal Society of Arts, London W.C.2. Mr. H. L. Kirke was in the Chair.

A discussion was held on " Concert Hall Acoustics ".

H. Bagenal opened the discussion.

5*th November* 1948

Extraordinary General Meeting, at Imperial College, London S.W.7. The President, Professor G. I. Finch was in the Chair.,

The following special resolution was passed :

" That as from January 1949, or from such date as the Council may see fit, the *Proceedings of the Physical Society* shall be issued in two parts, which parts shall be entitled respectively *Proceedings of the Physical Society A* and *Proceedings of the Physical Society B* ; and further, that each Fellow and Student Member shall be entitled to receive either *Proceedings A* or *Proceedings B* (whichever he elects) on payment of the ordinary current subscription, and shall be entitled to receive both parts of the *Proceedings* on payment of an additional annual subscription not exceeding the current Fellowship subscription, the exact amount to be determined by the Council in accordance with the Articles of Association › the Physical Society.

5th November 1948

Science Meeting, at Imperial College, London S.W.7. The President, Professor G. I. Finch, was in the Chair.

The following were elected to Fellowship, the first two being transferred from Student Membership : David Noel Ferguson Dunbar, Peter Ewart Watson, Maurice Evan Bell, Charles Alexander Cochrane, Sheila Marion Crawford, Frederick Paulinus Cunningham, Gerard Field, Louis Jacob, Dominic Johnston, George Guy Macfarlane, Harrie Stewart Wilson Massey, Archibald George Peacock, Cyril George Wilson, J. G. Raynor Young.

The following papers were read and discussed :

" The Scattering of Fast Beta-Particles through Large Angles by Nitrogen Nuclei ", by F. C. Champion and R. R. Roy;

" Beta Disintegration and Sargent Diagram ", by N. Feather and H. O. W. Richardson

" Investigations using a Permanent Magnet Double Beta-Ray Spectrograph with Coincidence Counting ", by N. Feather, J. Kyles and R. W. Pringle.

10th November 1948

The forty-second meeting of THE COLOUR GROUP, at the Lighting Service Bureau, London W.C.2. Dr. W. D. Wright was in the Chair.

The following papers were read and discussed :

" Measurement, Representation and Specification of Colour and Colour-rendering Properties of Light Sources ", by G. T. Winch and H. R. Ruff;

" Fluorescent Lamp Artificial Daylight Units for Colour Matching ", by G. T. Winch, W. Harrison and H. R. Ruff.

An informal report was given on the meeting of the Commission Internationale de l'Eclairage in Paris, July 1948, by members of the Colour Group who attended.

25th November 1948

The seventeenth meeting of THE LOW TEMPERATURE GROUP, at the Science Museum, London S.W.7. Sir Charles Darwin was in the Chair.

A lecture was given entitled " Irreversible Processes in Liquid Helium II ", by Professor C. J. Gorter.

25th November 1948

The eleventh meeting of THE ACOUSTICS GROUP, at Imperial College, London S.W.7. Mr. H. L. Kirke was in the Chair.

The following paper was read and discussed :

" Apparatus for Acoustical Measurements at Low Frequencies ", by R. S. Dadson and E. G. Butcher.

3rd December 1948

Extraordinary General Meeting, at Imperial College, London S.W.7. The President, Professor G. I. Finch, was in the Chair.

The following special resolution was passed :

" That Article 44 of the Articles of Association of the Society be altered as follows :—

Student Members shall be entitled to a copy of all publications issued by the Society upon such terms as the Council or a General Meeting of the Society may from time to time fix."

3rd December 1948

Science Meeting, at Imperial College, London S.W.7. The President, Professor G. I. Finch, was in the Chair.

It was announced that Council had elected the following to Student Membership : Peter John Cott, John Joseph Florentin, William Peter McKinney, Eric Shuttleworth, Donald Wesley Willis, Ronald Drayton Brown, Michael Onslow Bryant, Desmond Malone Burns, Ugo Camerini, Richard Hapgood Campbell, Peter Francis Chester, John Oswald Cope, Brian Arthur Evans, Cecil Bentley-Innes Glass, Stanley Hope, David John Ingram, Charles Hennie James Johnson, Aaron Klug, William Owen Lock, Ronald W. Maxwell, Hugh Muirhead, William Paterson Forsyth Raffan, Leonore Ritter, Michael Ian Scott.

The following were elected to Fellowship, the first-named being transferred from Student Membership : Raymond Jeffrey Slaughter, Mohammed Kashif Al-Ghita, Stanley Edgar Barden, Christopher Frederic Bareford, Albert Edward De Barr, David Robert Bates, Fathi Ahmed el Bedewi, Isaac Israel Berenblut, Arthur Cunliffe, A. Van Itterbeek, Max Krook, John Stewart Marshall, Bernard Wheeler Robinson.

The following papers were read and discussed :

" Refractive Index in Electron Optics and the Principle of Dynamics ", by W. Ehrenberg and R. E. Siday;

" Measured Properties of Strong ' Unipotential ' Electron Lenses ", by G. Liebmann.

8th December 1948

The forty-third meeting of The Colour Group, at Imperial College, London S.W.7. Mr. J. G. Holmes was in the Chair.

The following papers were read and discussed :

" The Scotopic Visibility Function ", by B. H. Crawford;

" Visual Purple and the Photopic Luminosity Curve ", by H. J. A. Dartnall.

9th December 1948

Science Meeting, at the Laboratories of Metropolitan-Vickers Electrical Co., Trafford Park, Manchester. The President, Professor G. I. Finch, was in the Chair.

The following papers were read and discussed :

" The Structure of Electro-Deposits ", by G. I. Finch;

" Observations on Electrical Behaviour of Silicon Carbide Contacts ", by E. W. J. Mitchell and R. W. Sillars;

" Structure and Electrical Properties of Surfaces of Semiconductors—Part I : Silicon Carbide ", by T. J. Jones, R. A. Scott and R. W. Sillars;

" Some Experiences in the Application of the Electron Microscope to the Study of Steels ", by J. Trotter and F. W. Cuckow;

" The Structure and Epitaxy of Lead Chloride Deposits formed from Lead Sulphide and Sodium Chloride ", by A. J. Elleman and H. Wilman.

16th December 1948

The twelfth meeting of The Acoustics Group, at Imperial College, London S.W.7. Mr. H. L. Kirke was in the Chair.

The following paper was read and discussed :

" Jet-propelled Aircraft Noise ", by S. C. Ghose.

17th December 1948

The thirty-fourth meeting of The Optical Group, at the Institute of Physics, London S.W.1. Professor L. C. Martin was in the Chair.

The following papers were read :

" On the Theory of Aplanatic Aspheric Systems ", by E. Wolf and G. D. Wassermann;

" A General Method for the Calculation of the Chromatic Variation of Lens Aberrations ", by H. H. Hopkins.

17*th December* 1948

The third Annual General Meeting of THE LOW TEMPERATURE GROUP, at the Science Museum, London S.W.7. Sir Charles Darwin was in the Chair.

The Minutes of the Second Annual General Meeting were read and confirmed.

The Report of the Committee for 1948 was presented.

The Officers and Committee for 1949 were elected.

Auditors were elected for the Special Fund.

The retiring President addressed the meeting.

Votes of thanks were recorded to the retiring Chairman and members of the Committee.

17*th December* 1948

The eighteenth meeting of THE LOW TEMPERATURE GROUP, at the Science Museum, London S.W.7. Sir Charles Darwin was in the Chair.

A discussion was held on " The Use of Thermodynamics Diagrams in Industry ". The subject was introduced by Professor D. M. Newitt.

14*th January* 1949

Science Meeting, at Imperial College, London S.W.7. The President, Professor G. I. Finch, was in the Chair.

It was announced that Council had elected the following to Student Membership : Jack Blitz, Brian Irvine Callin, John Anthony Champion, Robert Derek Craig, Gordon Arthur Dale, Cyril Henry Dix, Hector William Emerton, Donald Charles Field, Bernard Frank Figgins, Roy Sidney Gibbons, Donald Henry Grover, Raymond Frederick Hall, Leslie Jeffrey Hastewell, John Frank Hills, Walter Hopwood, John Arthur Hulbert, Arthur Langridge, Geoffrey Albert Lawrence, Albert John Manuel, Douglas Leonard Martin, Colyn Grey Morgan, Basil Leslie Morton, Edmund Gerard Muirhead, Frank Donald Stacey, Anthony Victor Stockley, John Thraves, Gerald Anthony Williams, John Francis Williams, Ronald John Wilson, Alan Neal Durham Young.

The following were elected to Fellowship, the first six being transferred from Student Membership : Patrick Bomyer, Sylvia Mary Gumbrell, Peter Donald Lomer, Lloyd Julian Perper, Michael Somerset Ridout, Krishnarao Govindarao Torgal, James Brooking Brown, Klaus Fuchs, Alfred Claude Jessup, John Moffatt, Frank Dudley Stott, James Harold Thorp, John Samuel Thorp, John Wilson.

The following papers were read and discussed :

" Travelling Wave Linear Accelerators ", by R. B. R.-Shersby-Harvie;

" Theoretical Design of Linear Accelerators for Electrons ", by W. Walkinshaw;

" Experimental Work on Corrugated Waveguides and Associated Components for Linear Electron Accelerators ", by L. B. Mullett and B. G. Loach.

26*th January* 1949

The thirteenth meeting of THE ACOUSTICS GROUP, at the Institution of Electrical Engineers, London W.C.2. Mr. H. L. Kirke was in the Chair.

A demonstration was given on " The Development of Magnetic Tape Recorders ", by B. E. G. Mittell.

The meeting was held jointly with the Radio Section of the Institution of Electrical Engineers.

4*th February* 1949

Science Meeting, at University College, London W.C.1. The President, Professor G. I. Finch, was in the Chair.

It was announced that Council had elected the following to Student Membership : Herbert Colin Bate, Alan E. Beck, Dennis Walter Bird, Walter Ernest Booth, James Graham Brown, Clifford Harry Champness, John Albert Chapman, David Michael Danziger,

Mehmet Yusuf Dizioğlu, Kenneth Thomas Dolder, Arthur James Dyer, Roland Ernest Ford, Clifford Gregory, Dorothy Joyce Harrington, Benjamin Peter Howorth, David William Shaper Limbert, Max Lipsicas-Lipschitz, John Morgan, Eric Norman-Wilson, John Osborne, Alexander Tudor Pomeroy, John Samuel Pyett, Robert Shannon, Roman Stefan Sidorowicz, Alan Smith.

The following papers were read and discussed :

" A Projection Model to Illustrate Crystal Structure with Surfaces of Misfit ", by E. N. da C. Andrade;

" The Mechanism of Dilatancy ", by E. N. da C. Andrade and J. W. Fox;

" The Effect of Pre-Strain on the Character of the Creep of Lead ", by E. N. da C. Andrade and A. J. Kennedy;

" The Thermal Etching of Single Crystals of Cadmium ", by E. N. da C. Andrade and R. F. Y. Randall;

" Viscosity and Density in the Supercooled Liquid State ", by C. Dodd and Hu Pak Mi.

12*th February* 1949

The forty-fourth meeting of The Colour Group, held jointly with the British Psychological Society, at Tavistock House, London W.C.1. Mr. J. G. Holmes was in the Chair.

A discussion was held on the techniques and methods of interpretation of experiments in colour as carried out by physicists, physiologists and psychologists. The discussion was introduced by W. A. Allen, R. C. Oldfield, R. W. Pickford, L. C. Thomson and W. D. Wright.

18*th February* 1949

The fourteenth meeting of The Acoustics Group, at Imperial College, London S.W.7. Mr. H. L. Kirke was in the Chair.

A discussion was held on the subject of " Applications of Ultrasonics ". Mr. G. Bradfield opened the discussion.

19*th February* 1949

Science Meeting, at Edinburgh University. The President, Professor G. I. Finch, was in the Chair.

The following papers were read and discussed :

" Experimental Tests of a Wide-Angle Beta-Particle Spectrometer using an approximately Prolate Spheroidal Magnetic Field ", by T. H. Braid and H. O. W. Richardson;

" Tests of a Prism Spectrograph of High Resolving Power ", by R. E. Siday and D. A. Silverston;

" An Emission Microscope for Photoelectron Autoradiography ", by (Miss) A. N. Barker, N. Feather and H. O. W. Richardson;

" The Use of Proportional Counters to Investigate Beta-Disintegration ", by S. C. Curran;

" Electrons as Nuclear Projectiles ", by B. Touschek;

" A new Approach to the Theory of Elementary Particles ", by M. Born and H. S. Green;

" Film Transport in Liquid Helium II ", by J. F. Allen;

" A new Photoelectric Amplifier ", by R. V. Jones;

" Long-Range Molecular Forces ", by J. Iball.

24*th February* 1949

The nineteenth meeting of THE LOW TEMPERATURE GROUP, at the Geological Society, London W.1. The meeting was held jointly with the Institution of Chemical Engineers. Professor F. E. Simon was in the Chair.

A Lecture was given on " Materials of Construction for use at Low Temperatures ", by E. W. Colbeck.

11*th March* 1949

Science Meeting, at the Science Museum, London S.W.7. The President, Professor G. I. Finch, was in the Chair.

It was announced that Council had elected the following to Student Membership : Herbert Roy Barnes, Clive Keith Coogan, Jack Dutton, Arthur Abraham Fröhlich, Brian Leonard Hart, Peter Arthur Lee, Colin Douglas McKenzie, Richard Leonard Mason, John Barlow Massey, Maurice George Mylroi, John Robert Pattison, Colin John Swanson, Peter Guy Towlson, Alan Reginald Watson, Donald Percy Dennis Webb, Kenneth Worthington.

The following were elected to Fellowship, the first thirteen being transferred from Student Membership : Edward Walter Bastin, Raymond Bowers, Richard Arthur Brown, Francis Maurice Comerford, Christian Ellis Coulman, James William Crawford, Robert Tudor Jarman, George Harry King, Eric Walter Lee, Charles Walter Morley, Michael William Ovenden, John Alexander Pryde, Geoffrey Cecil Pyle, Stanley Sumner Ballard, Max Born, Edward Colin Cherry, Claude Le Cointe, Ralph Forder Denington, Adam Wincety Gac, Hylton Judith Grenville-Wells, Arthur Ap Gwynn, Gordon Edward Hughes, Cecil Newton Kington, Anne Hutton Numbers, David Luther Phillips, Horace Hewitt Poole, Otto Pressel, Alec Radcliffe, Marion Amelia Spence Ross.

The following papers were read and discussed :

" Temperature Measurements of Flames containing Incandescent Particles ", by H. G. Wolfhard and W. G. Parker;

" A new Technique for the Spectroscopic Examination of Diffusion Flames at Normal Pressures ", by H. G. Wolfhard and W. G. Parker.

18*th March* 1949

The Second Annual General Meeting of THE ACOUSTICS GROUP, at the Royal Society of Arts, London W.C.2. Mr. H. L. Kirke was in the Chair.

The Minutes of the First Annual General Meeting were read and confirmed.

The Report of the Committee for 1948–49 was presented.

A proposal to make some alterations and additions to the Regulations of the Group was approved.

The Officers and Committee for 1949–50 were elected.

18*th March* 1949

The fifteenth meeting of THE ACOUSTICS GROUP, at the Royal Society of Arts, London W.C.2. Mr. H. L. Kirke was in the Chair.

The following paper was read and discussed :

" The Acoustics of Bells ", by Ir. E. W. van Heuven.

21*st March* 1949

Celebration of the Seventy-fifth Anniversary of the Foundation of THE PHYSICAL SOCIETY.

The thirty-third Guthrie lecture was delivered at the Royal Institution, London W.1. by Professor A. O. Rankine. His subject was " Experimental Studies in Thermal Convection ".

During the evening a Conversazione was held at the Rooms of the Royal Society.

23*rd March* 1949

The twentieth meeting of THE LOW TEMPERATURE GROUP, at the works of Messrs. Petrocarbon Ltd., Eccles, Manchester. Professor F. E. Simon was in the Chair.

An introductory lecture was given outlining the scope of the works and describing the low temperature plant. The pilot plant and the works were then inspected.

24*th March* 1949

The thirty-fifth meeting of THE OPTICAL GROUP, at Imperial College, London S.W.7. Professor L. C. Martin was in the Chair.

The following papers were read and demonstrations given :

" The Design of a Class of Variable Power Systems ", by H. H. Hopkins. The demonstration was given of a system with 2-inch aperture and 30-degree field for use with television cameras ;

" German Infantry Range-finders ", by E. Wilson.

30*th March* 1949

The ninth Annual General Meeting of THE COLOUR GROUP, at the Royal Photographic Society's rooms, London S.W.7. Mr. J. G. Holmes was in the Chair.

The Minutes of the eighth Annual General Meeting were read and confirmed.

The Report of the Committee for 1948–49 was presented.

The Officers and Committee for 1949–50 were elected.

A vote of thanks was given to retiring Committee Members.

30*th March* 1949

The forty-fifth meeting of THE COLOUR GROUP, at the Royal Photographic Society's rooms, London S.W.7. Dr. W. S. Stiles was in the Chair.

A discussion was held on Photoelectric Spectrophotometers and Tricolorimeters. The discussion was introduced by T. Vickerstaff.

The new Hilger Uvispek photoelectric spectrophotometer was demonstrated by Messrs. Hilger and Watts Ltd.

22*nd April* 1949

The sixteenth meeting of THE ACOUSTICS GROUP, at the National Hospital, London W.C.1.

The following paper was read and discussed :

" The Theory of Hearing ", by T. Gold and R. J. Pumphrey.

22*nd April* 1949

Science Meeting, at the Science Museum, London S.W.7. The President, Professor G. I. Finch, was in the Chair.

It was announced that Council had elected the following to Student Membership : Max Theodore Christensen, Alan Derek Harold Cripps, Francis Peter Edmund Gardner, Bernard Lionel Ginsborg, Henry Ralph Harman, Harvey Douglas Keith, Alan Bernard Lidiard, John Langley Rogers, Alfred Philip Tatt, Alan Teviotdale, Bernard John West.

The following were elected to Fellowship, the first-named being transferred from Student Membership : Werner Freitag, Henry Ian Allgood, Frank Foster Evison, Richard L. Extermann, Arthur J. Good, Pierre A. Grivet, Arnold Charles Lynch, Ian Mackay, Roy Middleton, Velimir Vook.

The twenty-fifth Duddell Lecture was delivered by Professor K. M. G. Siegbahn (Nobel Institute for Physics, Stockholm), whose subject was " On Gratings ".

27*th April* 1949

The twenty-first meeting of THE LOW TEMPERATURE GROUP, was held at the Laboratories of the British Oxygen Company, Morden, London S.W.19. Professor F. E. Simon was in the Chair.

An introductory lecture was given by P. H. Sykes. The work of the Development and Research Sections was outlined by P. M. Schuftan and A. M. Clark.

The laboratories were then inspected.

6*th May* 1949

Annual General Meeting, at the Royal Institution, Albermarle Street, London W.1. The President, Professor G. I. Finch, was in the Chair.

The Minutes of the Annual General Meeting held on 5th May 1948 were read and confirmed.

The Reports of the Council and the Honorary Treasurer and the Annual Accounts for 1948 were adopted.

The Officers and Council for 1949–50 were elected.

Votes of thanks were recorded to the retiring President, Officers and Members of Committees; to the Rector and Governing Body of Imperial College; the Director of the Science Museum; the Managers of the Royal Institution and the Royal Commission for the Exhibition.

6*th May* 1949

Science Meeting, at the Royal Institution, Albermarle Street, London, W.1. The President, Professor G. I. Finch, was in the Chair.

The following were elected to Fellowship, the first three being transferred from Student Membership : Jerzy Adam, Michael Julian Maurice Bernal, Stuart Astley Young, Arnold Maurice Dobson, Alan Sayed Farghaly, Frederick Charles Frank, Marjorie Elsie Pillow, Edmund James Pryor, John Douglas Swift, Harold William Taylor, Cyril Stanley Watt, Maurice Stafford Willis.

Prizes and Certificates awarded in the recent Craftsmanship and Draughtsmanship Competition were presented.

The Presidential Address was delivered by Professor G. I. Finch, whose subject was " Steam in the Ring Discharge ".

11*th May* 1949

The twenty-second meeting of THE LOW TEMPERATURE GROUP, at the Science Museum, London S.W.7. Professor F. E. Simon was in the Chair.

A discussion on " The Use of Thermodynamic Diagrams in Industry " was introduced by D. M. Newitt.

13*th May* 1949

The eighth Annual General Meeting of THE OPTICAL GROUP, at Imperial College, London S.W.7. Professor L. C. Martin was in the Chair.

The Minutes of the seventh Annual General Meeting were read and confirmed.

The Report of the Committee for 1948–49 was presented.

The Officers and Committee for 1949–50 were elected.

Votes of thanks were given.

13*th May* 1949

The thirty-sixth meeting of The Optical Group, at Imperial College, London S.W.7. Professor L. C. Martin was in the Chair.

The following paper was read :

" Notes on the Designing of Aspherical Magnifiers for Binocular Vision ", by C. E. Coulman and G. R. Petrie.

A lecture was given by M. André Maréchal of the Institut d'Optique.

20*th May* 1949

The seventeenth meeting of The Acoustics Group, at the Royal College of Organists, London S.W.7. Mr. H. L. Kirke was in the Chair.

The following paper was read and discussed :

" The Development of a new Electronic Organ ", by L. E. A. Bourne.

A demonstration was given.

2*nd June* 1949

Science Meeting, at the Science Museum, London S.W.7. The President, Professor S. Chapman, was in the Chair.

It was announced that Council had elected the following to Student Membership : David Spence Campbell, Reginald George Harlow, Roger George Jarvis, Derek Raynor, John Albert Bennett, Thomas Hamilton Braid, Norman Daniel Cowell, Kathleen Marjoria Gartwick, Michael Samuel Kisch.

The following were elected to Fellowship, the first two being transferred from Student Membership : Fritz K. Bowers, Cyril Scott, Frank William Cuckow, Antonius Mathias Johannes Friedrich Michels, Alexander John Moncrieff-Yeates, Leslie Walter Phipps, Walter Phipps, Denis Edwin Piper, Leonard Alfred Sayce, Harold Smith.

The twenty-sixth Duddell Medal, Prize and Certificate were presented to Dr. E. H. Land (Polaroid Corporation, Cambridge, Massachusetts), in recognition of his work on polarizing materials.

The twenty-sixth Duddell Lecture was then delivered by Dr. Land under the general heading of " A Colour Translating Ultra-Violet Microscope ".

10*th June* 1949

Science Meeting, at the Science Museum, London S.W.7. The President, Professor S. Chapman, was in the Chair.

The 1949 Thomas Young Oration was delivered by Mr. T. Smith, M.A., F.R.S., whose subject was " The Contributions of Thomas Young to Geometric Optics, and their Application to Present-day Questions ".

11*th June* 1949

The thirty-third meeting of The Optical Group, at Imperial College, London S.W.7. Professor L. C. Martin was in the Chair.

The following papers were read and discussed :

" The Computation of Large Numerical Aperture Telescope Objectives ", by T. Smith;

" The Boundary Wave Theory of Image Formation ", by L. C. Martin.

15*th June* 1949

The forty-sixth meeting of THE COLOUR GROUP, at the Institute of Ophthalmology, London, W.C.1. Dr. W. S. Stiles was in the Chair.

The following paper was read and discussed :

" The History of Artists' Pigments ", by H. J. Plenderleith.

14–16*th July* 1949

Science Meeting, at the Cavendish Laboratory, Cambridge. In the absence of the President, the meeting was opened by Professor G. I. Finch.

It was announced that Council had elected the following to Student Membership : John Robert Gabriel, Frederick John Hiorns, Bruce Sween Liley, John Alexander McDonnell.

The following were elected to Fellowship, the first two being transferred from Student Membership : Joan Edwards, William Leslie Wilcock, John Raymond Drabble, Oliver William Humphreys, Kamal Mikhail Matta, Clifford Charles McMahon, Margarita Eileen Monk-Jones, John Maurice McLean Pinkerton, Sidney Joseph Wyard.

Papers were read under the following headings :

The Regular Behaviour of Long and very Long Waves returned from the Ionosphere :

" The Regular Behaviour of Long and very Long Waves returned from the Ionosphere ", by J. A. Ratcliffe;

" Very Low Frequency Propagation ", by S. B. Smith and K. W. Tremellen;

" The Effects of Sky-Wave on the Planning of Navigational Aids using Frequencies in the 70–130 kc/s. Band ", by W. T. Sanderson;

" The Characteristics of Low-Frequency Radio Waves reflected from the Ionosphere, with particular reference to Radio Aids to Navigation ", by C. Williams;

" Measurements on Long and very Long Waves ", by R. N. Bracewell.

The Regular Behaviour of Medium and Short Waves :

" The Application of Ionospheric Data to Short-Wave Transmission Problems ", by W. J. G. Beynon;

" (P′, f) Records at Spitsbergen ", by A. B. Whatman.

The Irregular Behaviour associated with Solar Events :

" Irregular Behaviour of the Ionosphere associated with Solar Events ", by W. R. Piggott;

" Some Work at Cambridge on Radio Fade-outs ", by K. Weekes;

" Les renforcements brusques des ondes très longues ", by R. Bureau.

The Formation of Ionized Regions :

" The Formation of Ionized Regions ", by K. Weekes;

" Theoretical Considerations regarding the Formation of the Ionized Layers ", by D. R. Bates and M. J. Seaton.

Irregularities in the Horizontal Plane in the Ionosphere :

" Irregularities in the Horizontal Plane in Region E of the Ionosphere ", by J. W. Findlay;

" Meteor Ionization in the Upper Atmosphere ", by A. C. B. Lovell;

" Scattering of Radio Waves from Region E ", by G. Millington;

" The Variations in Direction of Arrival of High-Frequency Radio Waves ", by W. Ross;

" Diffusion des echoes su voisinage des fréquences critiques de F2 ", by R. Rivault.

REPORT OF COUNCIL FOR THE YEAR ENDED 31ST DECEMBER 1948

INTRODUCTORY AND GENERAL

1948 may in general be considered a satisfactory year for the Society. The membership has again risen considerably, the increase of the Fellows' annual subscription from two to three guineas having produced no serious losses. The financial position has improved, though publication costs continue to rise; the balance is still rather precarious, but steps have been taken which, it is hoped, will result in a further improvement in 1949.

The number of papers suitable for the *Proceedings* has so much increased that substantial changes in policy were necessary if undesirable delays in publication were to be avoided; it was therefore decided to divide the *Proceedings* into two Sections, A and B, each to be issued in monthly parts.

At the Annual General Meeting, Professor A. O. Rankine, who had been acting as Honorary Business Secretary, was succeeded by Mr. C. G. Wynne.

The continued generosity of the Royal Commission for the Exhibition of 1851 in providing accommodation for the Society's headquarters at 1 Lowther Gardens is once more gratefully acknowledged.

The Council again records the cordial thanks of the Society to the Rector and Governing Body of Imperial College and Professors Sir George Thomson and H. V. A. Briscoe for the great privilege of holding the Exhibition in the Physics and Inorganic Chemistry Departments of the College. For the use of the Lecture Theatres for Science Meetings of the Society and its four Groups the Council thanks the various bodies who gave them hospitality, in particular Sir George Thomson, Head of the Physics Department of Imperial College, the Director of the Science Museum, and the Managers of the Royal Institution.

MEETINGS

The Annual General Meeting was held at the Royal Institution on 5th May. Reports of the Council and the Treasurer and the Accounts and Balance Sheet for 1947 were presented and adopted, and Officers and Council for 1948–49 elected.

Three Extraordinary General Meetings were held during the year. The first, on 5th May, when Professor Ernest Orlando Lawrence was elected to Honorary Fellowship of the Society. The second, on 5th November, was held to consider the proposed division of the *Proceedings*. A Resolution was adopted that the *Proceedings* be issued in two Sections, A and B, and that each Fellow and Student Member be entitled to receive either Section A or Section B on payment of the ordinary current Annual Subscription, and to receive both Sections of the *Proceedings* on payment of an additional annual subscription to be determined by Council. At the third, on 3rd December, an alteration to Article 44 was approved, viz., that Student Members be entitled to a copy of all publications issued by the Society upon such terms as the Council or a General Meeting of the Society may from time to time fix.

A two-day summer meeting at the Clarendon Laboratory, Oxford, was devoted to microwave spectroscopy, and a full-day meeting at the Research Department of Metropolitan-Vickers Electrical Co. Ltd., Manchester, in December to surface structure and electrical properties of semiconductors. At one of the nine meetings held in London the Society was delighted to welcome one of its Honorary Fellows, Professor R. W. Wood, who lectured on spontaneous deformation of crystals.

MEMORIAL LECTURES AND AWARDS

Guthrie Lecture

The 32nd Guthrie Lecture was delivered at the Science Museum on 4th June by Sir George Thomson, who took as his subject " The Growth of Crystals ".

Charles Vernon Boys Prize

The 4th (1948) Prize was presented to Professor S. Tolansky at the Science Museum on 8th October, who delivered a discourse on his work entitled " Current Investigations with Multiple-Beam Interferometry ".

Holweck Prize and Medal

The 3rd (1948) award of the Holweck Prize of the Physical Society and Holweck Medal of the Société Française de Physique was to Professor Y. Rocard. The presentation took place at the Royal Institution on 5th May when Professor Rocard delivered a Holweck Discourse on " Sur les Conditions D'Auto-Oscillation des Systèmes Vibrantes ".

EXHIBITION OF SCIENTIFIC INSTRUMENTS AND APPARATUS

The 32nd Exhibition was held at Imperial College on 6th to 9th April. The attendance was higher than in 1947 (about 9,400 excluding holders of members' and exhibitors' passes). The Catalogue was available in good time, and all orders were fulfilled before the opening of the Exhibition.

PUBLICATIONS

Proceedings

During 1948 a start has been made in decreasing the period which elapses between the time of submission of a paper for publication and its appearance in print. This has been made possible in part by the increase in paper allocated to the *Proceedings* and in part by the raising of the standard of presentation required for papers accepted for publication. The institution of " Letters to the Editor " has met with approbation and is now a regular feature of the *Proceedings*. These letters are not intended to be substitutes for detailed publication of completed work, but for preliminary announcement of the results where these may be of importance even without details of the methods, or of proposals for work to be undertaken later, or they may give short outlines of proposed new methods of attack, free for others to take up. It is not desired in general that they should be the sole medium of publication of slight researches which the author judges to be unworthy of longer publication.

In order further to expedite publication, from January 1949 the *Proceedings* is to appear monthly in two sections. It will be recalled that proposals for establishing a journal of Applied Physics had been considered and rejected. It was next proposed that the *Proceedings* should be separated into papers which deal with pure physics, and others with technological physics, but examination showed the impracticability of this mode of division. It led in particular to the disadvantage that papers on a given subject—say the motion of electrons in a field—would be found in both parts, according to the point of view adopted in the treatment. It was felt that workers would prefer all papers on a given field to be in one part, whether they were written with applications in mind or not. In consequence, it has been decided that Section A shall cover atomic and sub-atomic physics, including such subjects as crystal structure, quantum mechanics and spectra, whilst Section B deals with macroscopic physics, including such subjects as acoustics, optical design and radio.

Each member of the Society receives one part, at his choice, and may purchase the other at a reduced price. The cost of supplying both parts to all members would have involved raising the subscription to more than 4 guineas, and was unacceptable to the General Meeting held on 5 November.

Reports on Progress in Physics

Volume **11** (1946–47) was published in July 1948 and is in great demand. Some delay was experienced in despatching of orders because of the difficulty of obtaining the standard green cloth for binding. There is a long waiting list for some earlier issues of the *Reports*, viz., Volumes **1–3** and **5–9**, which are out of print. The Society is anxious to buy second-hand copies of these Volumes.

Volume **12** (1948–49) is in active preparation and should be published during the summer of 1949.

Special Reports

During the year the publication of the following reports was completed: *Report on Colour Terminology*, prepared by a Sub-Committee of the Colour Group of the Society; *The Strength of Solids*, a report of a summer conference held at the H. H. Wills Physical Laboratory of the University of Bristol in July 1947; *The Emission Spectra of the Night Sky and Aurorae*, a report of a conference arranged under the auspices of the Gassiot Committee of the Royal Society.

In general the sales of Special Reports are disappointing, and the Council of the Society has decided that in future the production of such Reports shall be kept to a minimum. There will, however, be published two reports of Symposia of the Acoustics Group, arrangements for which are already in hand.

Science Abstracts

Physics Abstracts (Section A of *Science Abstracts*), which is issued as part of the Fellowship subscription, continues to increase markedly in size and cost, but the Society has decided to continue its issue to Fellows as part of the subscription at present.

REPRESENTATION ON OTHER BODIES

The following appointments of representatives in 1948 are reported:—

National Committee for Crystallography: Dr. H. Wilman.

Journal of Scientific Instruments, Advisory Committee: Mr. J. H. Awbery, Professor G. I. Finch, Dr. H. H. Hopkins, Mr. A. T. Pickles, Mr. E. W. H. Selwyn, Dr. A. C. Stickland, Dr. W. S. Stiles, Dr. W. D. Wright.

British Standards Institution Sub-Committee on Temperature Measurement: Dr. E. Griffiths.

Royal Society National Committee for Physics: Dr. A. C. Menzies.

Royal Society National Committee for Scientific Radio: Dr. W. J. G. Beynon.

Faraday Society, Colloid and Biophysics Committee: Professor J. T. Randall.

Organization Committee for Symposium on Metallurgical Applications of the Electron Microscope: Dr. A. G. Quarrell.

British Standards Institution—Committee on Nomenclature and Symbols: Dr. L. Hartshorn.

Board of the Institute of Physics: Dr. H. H. Hopkins, Dr. A. C. Menzies.

British Standards Institution—Acoustics Industry Committee: Mr. H. L. Kirke.

Science Abstracts Committee of Management: Mr. J. H. Awbery, Dr. A. G. Gaydon, Dr. A. C. Menzies, Dr. D. Roaf.

OBITUARY

The Council records with great regret the deaths of the following: Dr. Irena Gimpel, Mr. T. H. Littlewood, Mr. G. L. Overton, Sir Clifford Paterson, Professor A. F. C. Pollard, Dr. J. H. Shaxby, Mr. E. W. Smith, Professor S. W. J. Smith (*Fellows*); and Miss Margaret Weiss (*Student Member*).

MEMBERSHIP

As the following tables show, the membership of the Society continues to increase.

Roll of Membership		Hon. Fellows	*Ex-officio* Fellows	Fellows	Student Members	Total
Totals, 31st Dec. 1947		8	4	1469	384*	1865*
Changes during 1948	Newly elected	1		86 } 125	126	
	Transferred			39 }	39 } 67	
	Deceased			7 }	1 }	
	Resigned			56 } 65	27 }	
	Lapsed			2 }		
	Net increase	1		60	59	120
Totals, 31st Dec. 1948		9	4	1529	443	1985

* Amended figure.

The following is a summary of the membership during the past eight years :—

Year	1941	1942	1943	1944	1945	1946	1947	1948
Newly elected Student Members	51	96	74	52	68	53	102	126
Transfers from Student Membership	13	20	22	23	52	46	33	39
Newly elected Fellows	28	57	42	36	56	139	126	87
Net increase in Fellowship	6	57	36	55	80	165	137	60
Net increase in total Membership	37	140	73	63	75	160	179	120

GROUPS

Colour Group

The Group held its eighth Annual General Meeting for the presentation of the Committee's Report on the work of the Group in 1947–48. Mr. J. G. Holmes was re-elected as Chairman of the Group, Mr. R. G. Horner was elected as Honorary Secretary, and the Committee for 1948–49 was elected.

Nine Science Meetings, brief particulars of which are given in the *Proceedings**, were held during the year. One of them, on 13th October at Northampton Polytechnic, was held as a joint meeting with the Optical Group.

The Report of the Sub-Committee on Colour Terminology was published as a companion to the earlier Report on " Defective Colour Vision in Industry ".

Optical Group

At the Seventh Annual General Meeting, which was held at Imperial College on 30th April 1948, Professor L. C. Martin was re-elected as Chairman of the Group, Mr. E. W. H. Selwyn was re-elected as Honorary Secretary, and the Committee for 1948–49 was elected.

Five Science Meetings, brief particulars of which are given in the *Proceedings**, were held during the year. One of them, on 13th October at Northampton Polytechnic, was held as a joint meeting with the Colour Group.

Papers read at the meetings of the Group have been published in the *Proceedings*† and in the *Photographic Journal*.

Low Temperature Group

The Third Annual General Meeting of the Group was held at the Science Museum on 17th December 1948, when Professor F. E. Simon and Professor D. M. Newitt were elected as Chairman and Vice-President respectively. Dr. G. G. Haselden was re-elected as Honorary Secretary and the Committee for 1948–49 was elected.

There were six Science Meetings held during the year, brief particulars of which are given in the *Proceedings**. The one held on 13th February 1948 was a joint meeting with the Institution of Chemical Engineers at the Institution of Civil Engineers, and the one held on 5th April was a joint meeting with the Institute of Refrigeration.

Acoustics Group

The Second Annual General Meeting of the Group took place at the Royal Institute of British Architects on 8th April, when Mr. H. L. Kirke and Dr. A. Wood were re-elected Chairman and Vice-Chairman respectively, Mr. W. A. Allen and Mr. A. T. Pickles were re-elected as Joint Honorary Secretaries, and the Committee for 1948–49 was elected.

Nine Science Meetings, brief particulars of which are given in the *Proceedings**, were held during the year.

Membership

The membership of the four Groups on 31st December 1948 was as follows :—

	Colour	Optical	Low Temperature	Acoustics
Members of the Physical Society	113	189	45	69
Members of participating bodies	62	80	35	146
Members of subscribing firms and Societies	33	75	—	—
Other members	15	19	10	50
	223	363	90	265

* *Proc. Phys. Soc.*, 1948, **61**, vi–xv. † *Proc. Phys. Soc.*, 1948, **61**, 489, 494.

REPORT OF THE HONORARY TREASURER FOR THE YEAR ENDED 31ST DECEMBER 1948

The accounts have been prepared on the same lines as last year, and show an excess of income over expenditure of £1067 4*s.* 2*d.*, against a deficit in 1947 of £1992 7*s.* 8*d.*

All items of expenditure show increases due to the increased activities of the Society.

On the income side the increase in the rate of Fellows' subscriptions from two to three guineas has produced a considerably greater revenue, while receipts from the sale of publications show a general all-round improvement.

A publication grant of £1500 from the Royal Society (against grants totalling £1400 last year) and also a legacy of £500 from the will of the late William Lucas (Fellow 1891) are gratefully acknowledged.

For the first time in recent years the stocks of publications on hand at the opening and closing of the year have been included, thereby giving a more accurate indication of the operations in connection with publications.

Dividends and interest show a decrease, which is accounted for by the fact that during 1947 the Society sold £1000 3% Savings Bonds, while £400 Lancaster Corporation Stock was redeemed and the proceeds not re-invested. There is also a reduction in income due to nationalization of the railways. No change has taken place in the Society's investments during the year.

(Signed) H. SHAW,
Honorary Treasurer.

25th February 1949.

SPECIAL FUNDS

W. F. STANLEY TRUST FUND

	£	s.	d.		£	s.
Carried to Balance Sheet	259	0	0	£338 19*s.* 1*d.* British Transport Stock 1978/88	259	0
	£259	0	0		£259	0

DUDDELL MEMORIAL TRUST FUND

Capital

	£	s.	d.		£	s.
Carried to Balance Sheet	374	0	0	£400 3½% War Loan Inscribed "B" Account	374	0

Revenue

	£	s.	d.		£	s.
Prize	20	0	0	Balance on 31st December 1947	13	19
Certificate	1	0	0	Interest on War Loan	14	0
Balance carried to Balance Sheet	6	19	11			
	£27	19	11		£27	19

SPECIAL FUNDS (*contd.*)

CHARLES CHREE MEDAL AND PRIZE FUND

Capital

	£	s.	d.		£	s.	d.
ıce carried to Balance Sheet	1866	10	0	£784 4% Funding Stock	839	0	0
				£1500 2½% Consols	1027	10	0
	1866	10	0		1866	10	0

Revenue

	£	s.	d.		£	s.	d.
ıce carried to Balance Sheet	86	11	10	Balance on 31 December 1947	17	14	8
				Interest on Investments	68	17	2
	£86	11	10		£86	11	10

CHARLES VERNON BOYS PRIZE FUND

Capital

	£	s.	d.		£	s.	d.
ıce carried to Balance Sheet	900	0	0	£1132 16*s.* 10*d.* 2½% Consols "B" Account	900	0	0

Revenue

	£	s.	d.		£	s.	d.
ıce on 31 December 1947	15	10	3	Interest on Investment	28	6	4
	26	5	0	Balance carried to Balance Sheet	14	8	11
ificate	1	0	0				
	£42	15	3		£42	15	3

HOLWECK PRIZE FUND

Capital

	£	s.	d.		£	s.	d
ıce carried to Balance Sheet	575	0	0	£575 3% Defence Bonds	575	0	0

Revenue

	£	s.	d.		£	s.	d.
e and Certificate	107	12	6	Balance on 31st December 1947	8	13	8
enses	66	19	6	British Council contribution to expenses	25	0	0
ıce carried to Balance Sheet	4	6	8	Interest on Investments	20	5	0
				Sale of Investment	100	0	0
				Grant from General Fund	25	0	0
	£178	18	8		£178	18	8

ADDENBROOKE BEQUEST

Capital

	£	s.	d.		£	s.	d.
ıce carried to Balance Sheet	337	0	0	£384 6*s.* 7*d.* 2½% Consols "D" Account	337	0	0

Revenue

	£	s.	d.		£	s.	d.
ıce carried to Balance Sheet	26	5	0	Balance on 31st December 1947	16	13	0
				Interest on Investments	9	12	0
	£26	5	0		£26	5	0

LIFE COMPOSITION FUND ON 31ST DECEMBER 1948

	£	s.	d.
16 Fellows paid £10	160	0	0
1 Fellow paid £15	15	0	0
1 Fellow paid £15 15*s.* 0*d.*	15	15	0
17 Fellows paid £21	357	0	0
1 Fellow paid £25 4*s.* 0*d.*	25	4	0
1 Fellow paid £26 5*s.* 0*d.*	26	5	0
56 Fellows paid £31 10*s.* 0*d.*	1764	0	0
1 Fellow paid £33 12*s.* 0*d.*	33	12	0
1 Fellow paid £67 14*s.* 6*d.*	67	14	6
	£2464	10	6

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31ST DECEMBER 1948

1947 £	EXPENDITURE	£ s. d.	£ s. d.
	To		
1140	"Science Abstracts"		788 5 0
	Publications:		
	Stock 1st January 1948	6700 0 0	
4769	"Proceedings"	6861 14 4	
1205	"Reports on Progress in Physics"	2198 5 7	
2796	Special Publications	1277 8 7	
			17037 8 6
800	Postage on Publications and General Correspondence		935 15 10
402	Agenda Papers, Notices and Expenses at Meetings		397 3 3
26	Honoraria to Special Lecturers		26 5 0
3451	Secretarial and Clerical Assistance and Office Expenses		4415 3 0
25	Grant to Holweck Prize Fund		25 0 0
97	Furniture and Office Equipment		133 2 6
2793	Exhibitions:—Expenses (excluding office and overhead)		3936 12 10
	Balance, being excess of Income over Expenditure, carried to General Fund		1067 14 2
£17,504			£28,762 10 1

1947 £	INCOME	£ s. d.	£ s. d.
	By		
	Subscriptions:		
110	Entrance Fees	88 4 0	
2752	Fellows	4428 7 11	
211	Student Members	353 14 6	
31	Colour Group	34 17 0	
61	Optical Group	96 2 6	
12	Low Temperature Group	63 1 0	
47	Acoustics Group	144 14 0	
200	"Science Abstracts"	281 16 4	
			5490 17 3
	Publications:		
2788	"Proceedings"	2878 6 6	
1046	"Reports on Progress in Physics"	1939 7 7	
997	Advertisements in "Proceedings"	1432 17 3	
579	Special Publications	1833 0 3	
639	Cambridge Conference, contribution towards cost of Report	– – –	
1400	Grants from Royal Society	1500 0 0	
—*	Stock of Publications 31st December 1948	7500 0 0	
			17083 11 7
328	Interest on Investments		273 2 2
41	Transfer from Life Composition Fund of amounts paid by Fellows now deceased		62 10 0
4270	Exhibitions:—Receipts		5852 9 1
1992	Balance, being excess of Expenditure over Income, carried to General Fund		– – –
£17,504			£28,762 10 1

* The opening stock was not included in the Income and Expenditure Account last year but was taken direct to the General Fund.

BALANCE SHEET AS ON 31st DECEMBER 1948

Liabilities

	£ s. d.	£ s. d.
Sundry Creditors		1966 12 7
Life Compositions:		
As on 31st December 1947	2443 11 0	
Add Payments during year	83 9 6	
	2527 0 6	
Less Transfer to Income and Expenditure Account	62 10 0	
		2464 10 6
Subscriptions:		
Members	587 9 11	
Publications	2391 6 8	
		2978 16 7
Special Funds:		
W. F. Stanley Trust Fund	259 0 0	
Duddell Memorial Trust Fund	380 19 11	
Charles Chree Medal and Prize Fund	1953 1 10	
Charles Vernon Boys Prize Fund	885 11 1	
Holweck Prize Fund	579 6 8	
Addenbrooke Bequest	363 5 0	
		4421 4 6
General Fund:		
As on 31st December 1947	11013 1 5	
Add Lucas Legacy	500 0 0	
Balance of Income and Expenditure Account	1067 14 2	
		12580 15 7
		£24,411 19 9

We have audited the above Balance sheet and have obtained all the information and explanations we have required. We have verified the bank balances and the Investments. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the books of the Society.

KNOX, CROPPER & Co.,
Chartered Accountants

Spencer House, South Place, E.C.2.
25th February 1949.

Assets

	£ s. d.	£ s. d.
Investments at market value on 31st December 1939 or cost:—		
W. F. Stanley Trust Fund:		
£338 19 1 3% British Transport Stock 1977/88		259 0 0
Duddell Memorial Fund:		
£400 3½% War Stock		374 0 0
Charles Chree Medal and Prize Fund:		
£784 4% Funding Stock	839 0 0	
£1500 2½% Consols	1027 10 0	
		1866 10 0
Charles Vernon Boys Prize Fund:		
£1132 16*s.* 10*d.* 2½% Consols		900 0 0
Holweck Prize Fund:		
£575 3% Defence Bonds		575 0 0
Addenbrooke Bequest:		
£384 6*s.* 7*d.* 2½% Consols		337 0 0
General Fund:		
£2100 2½% Consols	1437 10 0	
£1750 3½% War Stock	1636 0 0	
£500 4% Funding Stock	535 0 0	
£2300 3% Defence Bonds	2300 0 0	
£1919 16*s.* 3*d.* 3% British Transport Stock 1977/88	1533 0 0	
		7441 10 0
		11,753 0 0
(Market value on 31st December 1948: £12,836.)		
Dividends due on Investments		64 1 5
Inland Revenue—Income Tax recoverable		16 12 10
Subscriptions due		151 11 6
Sundry Debtors		1013 16 8
Stock of Paper and Binding Material	1480 2 9	
Publications	7500 0 0	
		8980 2 9
Cash at Bank: Current Account	2255 2 10	
Deposit Account	95 15 11	
Cash in hand	81 15 10	
		2432 14 7
		£24,411 19 9

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